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On the "Logical Necessity" of a Uniform Rate of Profit in Sraffa's *Production of Commodities by Means of Commodities*

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Abstract
The paper discusses the analytical arguments put forward by Sinha to prove the purely "logical" necessity of a uniform rate of profit in Sraffa's *Production of Commodities by Means of Commodities* and hence the independence of that condition from any consideration concerning competition. We find that such arguments cannot be accepted. In fact, far from establishing the existence of a contradiction that would be implied by unequal sectoral rates o profit, it is Sinha's arguments that are self-contradicting. In particular, his main arguments are ultimately based on the idea according to which the average rate of profit of a given economic system is, and at the same time is not, independent from prices.

Keywords: Sraffa, uniform rate of profit, price equations, Sinha.

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I. Introduction[1]

In this paper we shall discuss in some detail a central aspect of that "new interpretation" of Sraffa's contribution to economic thought that Ajit Sinha has been putting forward over approximately the last ten years.

As is well known, the thesis that lies at the heart of Sinha's interpretation has to do with the condition, which can be found in Sraffa's *Production of Commodities by Means of Commodities*, of a uniform rate of profit. More in particular, it is Sinha's contention that, while in the case of Smith, Ricardo, and Marx that condition should be referred to the operation of competition among the owners of the means of production, this would no longer be true when we look at Sraffa's work. In this case, Sinha claims, the condition of a uniform rate of profit would be a pure "logical necessity" (see, e.g., Sinha and Dupertuis 2009a, p. 496) and would hence arise quite independently of that state of adjustment of the "quantity brought to market" to the "effectual demand" of commodities that, in Smith's teaching, is the state that competition constantly tends to produce.

Naturally, it is not easy to see how a "logical necessity" may arise with regard to such an important issue as that of the uniformity of the rate of profit for one particular economist and not for other economists belonging to the same "classical tradition" (Sinha 2019). This is not, however, the question we want to address. Nor shall we take up those "exegetical arguments" (Sinha 2012, p. 1327) by which Sinha attempts to show that, when introducing a uniform rate of profit, Sraffa did not *mean* to refer to the operation of competition.[2] We shall rather stick to what may be seen as the strictly analytical side of the question: namely, to the arguments produced by Sinha in order to show that, since the condition of a uniform rate of profit would already emerge in *Production of Commodities* as the unavoidable outcome of some principle that can be justified on purely logical grounds, Sraffa would not have *needed* to refer that condition to the operation of competition.[3]

So far, this side of the question has not been generally discussed as a separate issue from Sinha's "exegetical arguments", which, on account of the relevance of the aspects of Sraffa's thought involved, have understandably received the greatest attention.[4] As a result, the strictly analytical arguments put forward by Sinha in support of his thesis have not received open and detailed discussion, this being particularly the case for what he calls his "formal argument", first produced in 2016 and taken up again both in 2018 and in 2021. This seems in turn to have produced the impression that, in order to reject Sinha's interpretation, one should be prepared to subscribe to the entire view of Sraffa's work held by Sinha's oppo-

nents. Naturally, there would be no harm if this were indeed the case. Interpretative discussions often cannot be decided without taking a general position on the issue under scrutiny. But, although we totally agree with the view of Sraffa's work that underlies such reactions to Sinha's interpretation as Aspromourgos (2012), Fratini (2012), Kurz (2012), Levrero (2012 and 2019), Smith (2020), and Schefold (2021), we believe it is important to note that a close examination of Sinha's analytical arguments is sufficient to see that, as it stands, his interpretation cannot be accepted under any view of Sraffa's work: for such arguments are seriously flawed and, in most cases, self-contradicting.

A distinction must be made in this respect. Clearly, a contradiction which could be shown to arise from the assumption of unequal sectoral rates of profit and from accepted premises of Sraffa's analysis (with the obvious exclusion of the condition of free competition) would be proof of the "logical necessity" of a uniform rate of profit in *Production of Commodities*. In none of his arguments, however, is Sinha able to establish the existence of such a contradiction. In fact, when we do come across a contradiction in his arguments, this is due entirely to the way in which these arguments are built.

The discussion of the arguments put forward by Sinha in support of his thesis is organised in a roughly chronological order. In particular, we shall devote Sections II-V to the arguments presented by Sinha before his 2016's "formal argument", giving special attention to two arguments based on the comparison between what Sinha calls "equivalent" production systems. This will allow us to recall the elements of Sraffa's analysis that are essential for reconstructing Sinha's overall position and to see the contradiction underlying such a position in a relatively clear form. In Section VI we shall present some considerations on Sinha's views concerning the "physical nature" of the rate of profit and their possible connection with his central thesis. We shall come to Sinha's "formal argument" in the last two Sections. More in particular, in Section VII we shall argue that such an argument cannot be accepted (a formal proof is provided in the Appendix), while in Section VIII, which recapitulates our discussion and contains some concluding remarks, we shall show that this "formal argument" is in fact based on the same contradiction that can be found in Sinha's previous arguments.

II. Sectoral rates of profit and the average (or general) rate of profit

The main argument adduced by Sinha in support of his thesis before 2016 (Sinha and Dupertuis 2009a, p. 497)[5] can be reduced to four propositions. In Sinha's own presentation,

the first of these propositions is somewhat technical. In essence, however, what Sinha maintains is (i) that when the rate of profit is uniform across industries, the average rate of profit is the same in the real economic system and in what Sraffa calls its associated Standard system, while, when the rate of profit is not uniform, the equality between those two average rates no longer holds in general, though it may hold in some particular cases.[6] To this first proposition Sinha adds: (ii) that the difference between their respective average rates of profit "contradicts the property of equivalent systems"; and (iii) that, as a matter of fact, the real economic system and the Standard system are "equivalent". It is, then, from these three propositions taken together that Sinha concludes (iv) that "the sectoral rates of profit must be the same to assure that $R = R^*$", where by R and R^* Sinha denotes, respectively, the average rate of profit of the real economic system and the average rate of profit of the Standard system.

As we shall see, the difficulties with Sinha's original argument arise in connection with his second proposition, which is both wrong and incompatible with his first proposition. It will be convenient, however, to start by giving a rather close look precisely at this first proposition, which, with its comparison between the real economic system and its associated Standard system, is central not only to the present argument, but to practically all the arguments Sinha has advanced in support of his thesis.

In order to do so, let us consider the simple economic system producing iron, coal, and wheat by means of the following yearly productive operations:

90 t. iron	+	120 t. coal	+	60 q. wheat	+	3/16 labour	→	180 t. iron
50 t. iron	+	125 t. coal	+	150 q. wheat	+	5/16 labour	→	450 t. coal
40 t. iron	+	40 t. coal	+	200 q. wheat	+	8/16 labour	→	480 q. wheat
180 t. iron		285 t. coal		410 q. wheat				

With a single exception, this is the productive system that Sraffa introduces in § 25 of *Production of Commodities* and to which Sinha generally refers in his works. The difference lies in the fact that, in our discussion, we propose not to follow Sraffa (and Sinha) in assuming that the various quantities of iron, coal, and wheat appearing on the left-hand side of the above table are used only as means of production. We shall assume instead that those quantities include both such means of production and the subsistence wage that is paid to the workers at the beginning of the yearly productive cycle. This will allow us to identify the difference between the commodities appearing on the right-hand side of our table and the commodities appearing on the left-hand side with the surplus of the econo-

my. We shall assume in addition that land is not scarce, so that such a surplus goes entirely to profits. Without altering the substance of the question, we will thus be able to discuss Sinha's arguments avoiding the two clearly irrelevant cases he invariably refers to: namely, the case of null wages, and the case of wages given in terms of an absolutely unrealistic basket of commodities (the commodities entering into Sraffa's Standard commodity).[7]

Starting from the economic system we have just introduced, let us now imagine to construct another productive system in which the proportions between the three industries may change, while the technical coefficients characterising each industry remain the same. Our aim is to obtain an imaginary system in which "the various commodities are represented among its aggregate means of production *in the same proportions* as they are among its products" (Sraffa 1960, p. 19), i.e., the Standard system associated to our original economic system. We shall come back to the role of such an imaginary system in a moment. For the time being, let us add two observations which may help to understand the specific features of the Standard system that appears in Sinha's arguments. It is clear in the first place that any commodity appearing among the products of a given real economic system without being used either as a means of production or for workers' subsistence could not be found in the Standard system associated to that system. The same applies to those commodities that, although used as means of production, do not enter directly or indirectly into the production of *all* the commodities in the system. It is only those commodities for which this is true (Sraffa's basic commodities), and necessarily all such commodities, that will be found in the Standard system. Secondly, it is important to note that, once its scale is fixed, the Standard system associated to a given real economic system is unique. Thus, if we decide with Sinha to keep the total amount of labour employed unchanged, we obtain from our original economic system — which, as can be seen immediately, contains only basic commodities — the following Standard system:

$$
\begin{array}{rcrcrcrcl}
120\text{ t. iron} & + & 160\text{ t. coal} & + & 80\text{ q. wheat} & + & 4/16\text{ labour} & \rightarrow & 240\text{ t. iron} \\
40\text{ t. iron} & + & 100\text{ t. coal} & + & 120\text{ q. wheat} & + & 4/16\text{ labour} & \rightarrow & 360\text{ t. coal} \\
40\text{ t. iron} & + & 40\text{ t. coal} & + & 200\text{ q. wheat} & + & 8/16\text{ labour} & \rightarrow & 480\text{ q. wheat} \\
\hline
200\text{ t. iron} & & 300\text{ t. coal} & & 400\text{ q. wheat} & & & &
\end{array}
$$

If now we assume a uniform rate of profit, we can write down, both for our real economic system with subsistence wages and for its associated Standard system, a system of price equations which is in every respect like the one that can be found in § 4 of *Production of Commodities*:

$$\begin{aligned}(90p_i + 120p_c + 60p_w)(1 + r) &= 180p_i \\ (50p_i + 125p_c + 150p_w)(1 + r) &= 450p_c \\ (40p_i + 40p_c + 200p_w)(1 + r) &= 480p_w\end{aligned} \qquad [1a]$$

$$\begin{aligned}(120p_i + 160p_c + 80p_w)(1 + r) &= 240p_i \\ (40p_i + 100p_c + 120p_w)(1 + r) &= 360p_c \\ (40p_i + 40p_c + 200p_w)(1 + r) &= 480p_w\end{aligned} \qquad [2a]$$

As can be seen, if we take one of the commodities as our numeraire, what we have here are two systems of three equations in the same three unknowns (the rate of profit and two relative prices). On account of the way in which the Standard system has been derived from the real economic system, the two systems will give in addition the same solution for their unknowns (as Sraffa 1960 puts it, the two systems are formed with "the same basic equations", only taken "in different proportions", p. 23). In particular, the uniform rate of profit is in both systems equal to 20%, while, if we take wheat as our numeraire, the price of iron is equal to 11/3 and the price of coal is equal to 4/3.

It can then be seen that, when the rate of profit is uniform, the average rate of profit will be the same in the real economic system and in its associated Standard system: for the average rate of profit is the weighted average of the sectoral rates of profit and will thus be equal to such rates whenever they are uniform across the industries.

To see what happens with different sectoral rates of profit, we must first of all consider that the weights used to determine the average rate of profit are given by the ratio between the value of the capital that is advanced in each industry and the value of the capital that is advanced in the whole economic system. Using Sinha's symbols introduced above, we can thus write the following two equations:

$$R = [(90p_i + 120p_c + 60p_w)\, r_i + (50p_i + 125p_c + 150p_w)\, r_c + (40p_i + 40p_c + 200p_w)\, r_w]/(180p_i + 285p_c + 410p_w) \qquad (1)$$

$$R^* = [(120p_i + 160p_c + 80p_w)\, r_i + (40p_i + 100p_c + 120p_w)\, r_c + (40p_i + 40p_c + 200p_w)\, r_w]/(200p_i + 300p_c + 400p_w) \qquad (2)$$

As can be seen, in both systems the average rate of profit can also be expressed as the ratio between the value of that part of the social product that goes to profits and the value of the capital that is advanced in the whole economic system: that is, as the general rate of profit. Instead of our previous equations, we can thus write the following two equations:

$$R = (165p_c + 70p_w)/(180p_i + 285p_c + 410p_w) \qquad (1')$$

$$R^* = (40p_i + 60p_c + 80p_w)/(200p_i + 300p_c + 400p_w) \qquad (2')$$

The difference between the real economic system and the Standard system becomes now apparent. It can be seen at once that while equation (1′) cannot be solved without first knowing prices, this can be done in the case of equation (2′). The reason is that, in the Standard system, the general rate of profit, and hence the average rate of profit, is given by the ratio between two different amounts of the same composite commodity (the Standard commodity, in which, for every ton of iron, we find 1,5 tons of coal and 2 quarters of wheat). Naturally, under our assumption concerning wages, the relevance of this result lies in the fact that what can be determined without expressly taking prices into consideration is the rate of profit that can be uniformly paid in the real economic system. After all, the Standard system is just an imaginary system and we are not interested in such a system for its own sake. But, with regard to Sinha's first proposition, what is important in this physical determination of the average rate of profit of the Standard system is that it shows that, in our example, such a rate would remain equal to 20% even if we were to introduce in that system "an arbitrary set of prices that gives different rates of profit for different sectors" (Sinha and Dupertuis 2009a, p. 497). Since the same cannot be said of the average rate of profit of the real economic system (for the latter is not independent of prices), we can thus be sure that, when prices do not assure a uniform rate of profit, the average rate of profit cannot *always* be the same in the real economic system and in the Standard system.[8]

There is, then, just one point that remains to be clarified in connection with Sinha's first proposition. As we have seen, what is stated in that proposition is that, when the rate of profit is not uniform, the average rate of profit will not *in general* be the same in those two systems. Indeed, Sinha hints at a "highly unlikely but mathematically possible scenario" (Sinha and Dupertuis 2009a, p. 497) in which that equality would hold without implying a uniform rate of profit. Now, it is easy to see how this may happen when the real economic system includes commodities that are not used either as means of production or for workers' subsistence. As we have seen, such commodities will not appear in the Standard system, so that there will certainly be cases in which their prices can be taken so as to smooth out a difference between the average rate of profit of the Standard system and the average rate of profit obtained by looking exclusively at the basic industries of the real economic system. But the "scenario" referred to by Sinha does not depend on the existence of non-basic commodities. This can be seen by deriving from equation (1′) the condition that pric-

es must satisfy in order for the average rate of profit to be equal to 20% in our real economic system. This gives us the following equation:

$$p_i = 3\, p_c - 1/3 \qquad (3)$$

As we shall see in connection with his "formal argument", Sinha seems to be unaware not just of the exact formulation of equation (3) but more generally of the fact that his "mathematically possible scenario" implies a restriction of some sort on prices. From our present point of view, however, equation (3) is sufficient to show that, as stated in Sinha's first proposition, in addition to the prices that assure a uniform rate of profit there might be a whole set of systems of positive prices for which the average rate of profit is the same in the real economic system and in its associated Standard system, and that, at the same time, such a set of price systems is a negligible subset of the set of admissible prices.[9]

III. The "property of equivalent systems" and the contradiction in Sinha's position

In his works devoted to the interpretation of Sraffa's thought, Sinha never gives an explicit definition of "equivalent systems". We can nevertheless accept his third proposition stating that a given real economic system and its associated Standard system are "equivalent". Indeed, at least in the case of basic-good systems, it is sufficiently clear from his use of the expression that, according to Sinha, two production systems are "equivalent" whenever it is possible to move from one system to the other by means of a change in the proportions among their industries that leaves both the total amount of labour employed and the technical coefficients of production unchanged. It must be said, then, that Sinha provides us with a criterion for recognising what he sees as "equivalent systems": and there can be no doubt that the criterion is satisfied when we look at the real economic system and at the Standard system we have introduced in Section II. To this we must add that, as we know, those two systems lead to the determination of the same relative prices and the same uniform rate of profit, so that they are in fact equivalent *in this respect*.[10]

Before we proceed, we may see how what we have just said allows us to present the exact formulation given by Sinha to his first proposition. In order to do so, let us consider what Sinha calls a "class of equivalent systems" (Dupertuis and Sinha 2009b, p. 1072), i.e., all the possible reallocations of the total amount of labour employed in the real economic system to its basic-good industries that leaves the technical coefficients of production unchanged. Now, it can be seen at once that all the production systems belonging to the same "class" will be associated to the same Standard system and that, when the rate of profit is uniform,

the average rate of profit will be the same in all such systems. As we have seen in the previous Section, for any one of such systems we may be able to identify prices which, though not implying a uniform rate of profit, assure that the average rate of profit is the same in *that* particular system and in the Standard system associated with the "class of equivalent systems". But it is clear that the intersection between all such sets of price systems contains only the single price system that assures a uniform rate of profit. The formulation of Sinha's first proposition that we have seen in Section II can thus be seen as a particular case of the more general (though hardly more informative) formulation given by Sinha himself: the average rate of profit is the same in "all the possible imaginary reallocation [sic] of the total labour of the real system" — i.e., in all production systems that are "equivalent" to such system — "if and only if all the sectoral rates of profit are equal" (Sinha and Dupertuis 2009a, p. 497).

That said, we are finally prepared to see the difficulties that arise in connection with Sinha's second proposition: namely, the proposition according to which the difference in their respective average rates of profit "contradicts the property of equivalent systems". Now, what is certainly true in this respect is that, since "equivalent" production systems share the same technical coefficients for their basic industries, the rates of profit for these industries will be the same in all such systems whenever prices are the same. The fact is, however, that this is not in general sufficient to determine the average rate of profit of a given production system. This is immediately evident if one thinks of a real economic system with non-basic products and unequal sectoral rates of profit. But even in the case of basic-good systems, the average rate of profit will not in general depend only on the sectoral rates of profit. Save for the case of a uniform rate of profit, such an average rate will depend in addition on the proportions between the different industries of the particular system for which that average rate is being determined: and, as we know, such proportions are precisely what under Sinha's criterion is changing when passing from a given production system to an "equivalent" system. It is easy to see, then, that, contrary to what is stated in Sinha's second proposition, the average rate of profit can be different in different production systems without this being in any way in contradiction with what has led us to group such systems in the same "class of equivalent systems".

This leads us to a further consideration. As we know, given two production systems employing the same amount of labour, it is by looking exclusively at their technical conditions of production that, according to Sinha's criterion, we should decide whether those systems

are "equivalent" or not. It follows, then, that, whatever property "equivalent" systems might share, this must be a property that depends on their technical conditions of production alone. But what this means is that, in order for Sinha's second proposition to be true, the average rate of profit of *any* production system, and not only that of its associated Standard system, should be *independent of prices*. It can then be seen that the main argument Sinha has initially adduced in favour of the "logical necessity" of a uniform rate of profit is not only based on an error in one of its propositions. Sinha's argument appears to be based in addition on a rather evident contradiction between two of its propositions. For, as we have just seen, by asserting with his second proposition that the average rate of profit cannot be different in "equivalent systems", Sinha implies that such an average rate is independent of prices in any production system; whereas the fact that the average rate of profit generally depends on prices is clearly implied in his first proposition, where we are correctly told that, in any given "class of equivalent systems", such an average rate will be the same *if and only if* prices are such as to assure a uniform rate of profit.

IV. The maximum rate of expansion

As we shall see, Sinha's belief in an average rate of profit that would be given independently of prices in the real economic system does not appear only in the argument we have discussed so far. Before we come back to this central aspect of his position, it will be useful however to give a quick look at a secondary argument which Sinha sometimes draws from his notion of "equivalent systems" and which does not involve the average rate of profit (Sinha 2009, p. 57, restated in Sinha 2014, p. 87).

Formally, the argument in question is rather close to the one we have discussed in the previous two Sections. Indeed, what Sinha maintains in this case is, on the one hand, that, since the real economic system and the Standard system are what he calls "equivalent systems", "the maximum rate of expansion of the two systems must be equal"; and, on the other hand, that such an equality holds "if and only if the sectoral rates of profit are equal in both systems".

Now, it is easy to see that, whenever such a rate can be properly defined,[11] the maximum rate of expansion is always the same in the real economic system and in its associated Standard system. This happens, however, precisely because, unlike the average rate of profit, the maximum rate of expansion of any production system depends exclusively on the technical conditions of production that the real economic system and the Standard system

have in common. There is, then, no set of prices, and hence no combination of the sectoral rates of profit, for which the maximum rate of expansion could possibly be different in the real economic system and in the Standard system. To see this, suppose that, in the real economic system we have considered so far, technical progress is absent and constant returns to scale prevail. Suppose in addition that the entire surplus is used for investment. It can then be shown (see, e.g., Pasinetti 1977) that, in order for this economic system to grow at its maximum rate of expansion, its industries must first be reproportioned so that each commodity can be found in the same proportions among its means of production and among its products. In other words, such an economic system will grow at its maximum rate of expansion only if it is first *actually* transformed into that *imaginary* system that in Sraffa's analysis is the Standard system.

We can see, then, that in this case too Sinha's argument is not only based on an error in one of its propositions, but also on an obvious contradiction between two of its propositions: for Sinha is in fact claiming that the maximum rate of expansion of the real economic system is, and at the same time is not, independent of prices.[12]

V. The "mathematical property of the average"

Let us go back to Sinha's ideas concerning the average rate of profit of the real economic system. In Section III we have shown how Sinha's belief in the independence of such an average rate from prices can be inferred from his appeal to a "property of equivalent systems" that would be contradicted by such an average rate being different from the average rate of profit of the Standard system. However, the most striking manifestation of such a belief is probably represented by a particular procedure Sinha often resorts to in order to determine the numerical value of the average rate of profit of the real economic system.

The procedure, which we are told has to do with the possibility of "equally distribut[ing]" an average "over the whole population" (Sinha 2014, p. 85), was first introduced in Sinha's contribution to the Symposium on Sraffa's thought organised by the *Cambridge Journal of Economics* in 2012. In this case, Sinha starts by dropping the condition of a uniform rate of profit from our equation systems [1a] and [2a]. He thus obtains the following systems of equations:

$$\begin{aligned} (90p_i + 120p_c + 60p_w)(1 + r_i) &= 180p_i \\ (50p_i + 125p_c + 150p_w)(1 + r_c) &= 450p_c \\ (40p_i + 40p_c + 200p_w)(1 + r_w) &= 480p_w \end{aligned} \qquad [1b]$$

$$\begin{aligned}(120p_i + 160p_c + 80p_w)(1 + r_i) &= 240p_i \\ (40p_i + 100p_c + 120p_w)(1 + r_c) &= 360p_c \\ (40p_i + 40p_c + 200p_w)(1 + r_w) &= 480p_w\end{aligned} \qquad [2b]$$

As can be seen, if we take one of the commodities as our numeraire, what we have here are two systems of three equations in the same five unknowns (two relative prices and the three sectoral rates of profit). As such, these systems cannot be solved. We know nonetheless that the average rate of profit associated to equation system [2b] is equal to 20%: for we can determine this average rate from equation (2′) without knowing prices. The crucial step in Sinha's reasoning consists then in asking whether something similar holds for the real economic system as well: in asking, that is, whether, confronted with equation system [1b], we can "determine the average rate of profit of the global [sic] system (i.e. R) from the given information" (Sinha 2012, p. 1327).

According to Sinha, "[t]he answer is yes". This would happen, he claims, because "if we *assume* that all the rates of profits were equal, then it reduces the number of unknowns to three and we could determine this rate of profit, which must be equal to the average rate of profit R" (p. 1328; italics in the original). Now, it should be clear that what Sinha believes he has determined with his "assumption" is not the average rate of profit that is established in the real economic system *when* the rate of profit is uniform, but the average rate of profit that would be established in that system *in any case*. Indeed, not only has Sinha explicitly claimed to rely on nothing else than the "given information" embodied in system [1b]; he also urges the reader to note

> that we do not know whether this average rate of profit is equally or unequally distributed in the system and, therefore, we still do not know whether the prices associated with an equal rate of profits hold in the given real system or not.

As can be seen, in his comment Sinha makes no reference to those particular price systems (which we have seen at the end of Section II) that, though giving unequal sectoral rates of profit, ensure all the same that the average rate of profit of our real economic system is equal to the average rate that is established in this system when the rate of profit is uniform. What Sinha is claiming is, in fact, that the determination of the average rate of profit does not result in any restriction on prices, which, clearly, would only be possible if that average rate were independent of prices. The procedure we have just seen can therefore be described as the determination of the numerical value of the average rate of profit of the real economic system by means of what we may call the purely instrumental assumption of a uniform rate of profit.

It must be noted, however, that the possibility of "equally distribut[ing]" an average "over the whole population" seems to have at the same time a different role in Sinha's reasoning: namely, that of *proving* the uniqueness of the average rate of profit of the real economic system, and hence its necessary equality with the average rate of profit of the Standard system. At least this is what one is led to believe when, on the one hand, Sinha asserts that "whatever turns out to be the average rate of profit [of the real economic system], the mathematical property of the average ensures that it can be equally distributed over the total capital" (Sinha 2012, pp. 1327-1328; see also Sinha 2021, p. 18), and, on the other hand, he notices that, if for given prices the average rate of profit of our real economic system is different from 20%, then "when we try to distribute this 'average' rate of profits equally to all the industries we find that the system breaks down" (Sinha 2014, p. 85).

But it is easy to see that the "break down" of the system Sinha refers to contradicts no "mathematical property of the average" and hence does not allow to conclude that no average rate of profit other than 20% is possible in our real economic system. The fact is that, contrary to what Sinha seems to believe, the question he is really addressing is not that of assigning alternative distributions to a given average, but that of obtaining the same average rate of profit with different price systems. Indeed, Sinha starts with an average rate of profit resulting from prices which do not entail a uniform rate of profit and then tries to obtain that same average rate of profit from prices entailing a uniform rate of profit. What Sinha describes as an attempt at "equally distributing" a given average is, therefore, an attempt at finding, among the admissible price systems for equation system [1b], more than one price system that is compatible with the uniformity of the rate of profit. And, once the question is presented in these terms, it comes as no surprise that the average rate of profit that can be generated by equal sectoral rates is unique. The fact that "[t]he Standard average is the only average that can be distributed equally across the industries" (Sinha 2021, p. 23) has therefore no bearing on the possibility of different average rates of profit associated with unequal sectoral rates.

Be it a consequence or the intended proof of the idea of an average rate of profit that would be given in the real economic system independently of prices, the procedure by which Sinha determines the average rate of profit of the real economic system by means of the assumption of a uniform rate of profit implies the average rate of profit to be always the same in the real economic system and in the Standard system. Sinha is thus led back to the necessity of deriving a restriction on the behaviour of the sectoral rates of profit from an

equality that in his opinion would hold independently from such behaviour. And it is for this reason that, after claiming he has determined the average rate of profit of the real economic system without knowing "whether this average rate of profit is equally or unequally distributed", Sinha (2012, p. 1328) is once again forced to admit that such an average rate is in fact *not* independent of the sectoral rates of profit: "[i]t is clear that R will always be equal to R^* [...] if and only if, all the r values in equation system [1b] must [sic] be equal".

VI. The "physical nature" of the rate of profit

Before coming to his "formal argument", we may take a brief pause in our discussion of Sinha's arguments. Although the question does not ultimately concern their validity, it may be useful to inquire into the possible origin of the idea, which as we have seen plays a major role in such arguments, of an average rate of profit that would be given in the real economic system independently of prices.

Now, there can be little doubt that such an origin must be sought in Sinha's views concerning "the physical nature of the rate of profits" (Sinha 2012, p. 1336). After all, it is Sinha himself who is quite clear on this point when he writes (Sinha 2016, p. 202):

> [T]he average rate of profits is a physical property of the actual system and is completely independent of prices or the condition of a uniform rate of profits across industries. "Even if" prices were such that the industrial rates of profit were not equal, the average rate of profit would not be disturbed.

But the connection between Sinha's views on the "physical nature" of the rate of profit and his ideas on the independence of the average rate of profit of the real economic system from prices is not always as direct as it appears from this passage. Indeed, the possibility of referring to a "physical rate of profits" of the real economic system (Sinha and Dupertuis 2009a, p. 497 and Sinha 2010, p. 291) more often emerges in Sinha's work in the course of less unambiguous arguments as the one contained in the following passage:

> [T]he aggregate or the *global* rate of profit of the empirical or the real system of production is given in terms of a ratio of heterogeneous goods. Since it is a ratio of heterogeneous goods, its value is unknown.
>
> In the above given example, if the aggregate or the global rate of profit of the system is given by R, then the value of $(1 + R)$ = (180 t. iron + 450 t. coal + 480 qr. wheat)/(180 t. iron + 285 t. coal + 410 qr. wheat). Now, if we multiply the physical amounts of iron, coal and wheat by taking several arbitrary prices of iron, coal and wheat, we would find that the value of the above given ratio will change with changes in prices. However, since the physical ratio remains the same, it immediately tells us that prices can create a "nominal" effect on R (a sort of optical illusion) which is completely independent of its physical value.[13]

As can be seen, in the second passage Sinha does not claim that the average rate of profit of the real economic system cannot be "disturbed" by prices. On the contrary, he openly admits the existence of such "disturbances", though interpreting them as mere "optical illusions". This happens because Sinha suggests the possibility of drawing a particular distinction with respect to the average rate of profit of the real economic system. This is the distinction between, on the one hand, what we may call a more visible, though at the same time more superficial, expression of such an average rate, which is represented by its "value" and which "will change with changes in prices", and, on the other hand, what would be a more hidden expression of that same average rate, which would somehow be given "in terms of a ratio of heterogeneous goods".

It should be noted incidentally that, if Sinha's proposed distinction could be accepted, his thesis concerning the nature of the condition of a uniform rate of profit would have to be significantly altered. Indeed, the role that that distinction would lead us to ascribe to the condition of a uniform rate of profit is that of dispelling the "optical illusion" represented by the influence that prices may have on the average rate of profit of the real economic system; and this is clearly quite a different matter from saying that the condition of a uniform rate of profit is a "logical necessity". However, what is really important from our point of view is that Sinha's distinction cannot be accepted. As is clear from the passage we have just quoted, what Sinha presents as the more hidden expression of the average rate of profit of the real economic system is nothing but the ratio between the commodities that make up the surplus of that system and the commodities that make up its capital. Now, such a ratio can only tell us how many units of the composite commodity forming the surplus of the real economic system can be found in that system for each unit of the *different* composite commodity forming its capital. It is not, then, a rate of profit, as can be seen for example by considering that the numerical value of such a ratio will change with changes in the physical unit by which those two composite commodities are measured. Such a "technical ratio" (Sinha 2012, p. 1329) can then neither directly establish a particular role for the condition of a uniform rate of profit, nor indirectly support the idea according to which the average rate of profit must be the same in the real economic system and in the Standard system.

VII. Sinha's "formal argument"

The special interest of Sinha's "formal argument" is due not only to the importance that Sinha himself clearly attaches to it, but also to a significant difference such an argument has

with respect to the arguments we have seen so far. As in the previous cases, Sinha tries to establish the "logical necessity" of a uniform rate of profit by showing that unequal sectoral rates of profit would involve a contradiction. In order to do so, however, he no longer relies on some "property" whose exact nature has in turn to be established. On the contrary, Sinha's aim is now to present a case in which the existence of a contradiction implied by the assumption of unequal sectoral rates of profit would be immediately visible when comparing the real economic system and its associated Standard system.

Sinha's "formal argument" was originally presented in the Appendix to chapter 7 of his 2016 *A Revolution in Economic Theory: The Economics of Piero Sraffa* by means of a new formulation of equation system [1b]. What distinguishes this new formulation is that the sectoral rates of profit, instead of appearing directly as such, are now expressed as *deviations* from the average rate of profit. We thus have the following system of four equations, where the first three equations express the sectoral rates of profit as deviations from a certain number R, while the last equation states that that number is equal to the average rate of profit of the system under discussion:

$$\begin{aligned}
&(90p_i + 120p_c + 60p_w)\,(1 + R + \lambda_i) = 180p_i \\
&(50p_i + 125p_c + 150p_w)\,(1 + R + \lambda_c) = 450p_c \\
&(40p_i + 40p_c + 200p_w)\,(1 + R + \lambda_w) = 480p_w \\
&(90p_i + 120p_c + 60p_w)\lambda_i + (50p_i + 125p_c + 150p_w)\lambda_c + (40p_i + 40p_c + 200p_w)\lambda_w = 0
\end{aligned} \qquad [1c]$$

Given this system of equations, the contradiction Sinha has in mind would emerge when, for a given set of prices (and hence for a given set of λ's), one wished to move from the real economic system to the Standard system: for, Sinha claims, with non-zero λ's and hence with unequal sectoral rates of profit, in the passage from one system to the other we would have both a reason to expect the average rate of profit to change and a reason to expect such an average rate not to change.

As for the expected change in the average rate of profit, this can be seen according to Sinha (2016, p. 223) first by assuming "[w]ithout loss of generality" that the rate of profit is above the average rate in the iron industry and below the average rate in the coal and in the wheat industry — that is to say, "that $\lambda_i > 0$ and λ_c and $\lambda_w < 0$" — and next by recalling how the Standard system was built from the real system. In order to obtain the Standard system we have used so far, Sinha reminds us, "we have increased the relative weight of the iron industry and reduced the relative weight of the coal industry with the wheat industry's weight remaining the same". Thus, Sinha claims, since we are expanding the industry obtaining a

higher-than-average rate of profit, and contracting or leaving unchanged the industries obtaining a lower-than-average rate of profit, "one must expect the average rate of profits of this system of equations to rise".[14] But why should we expect at the same time the average rate of profit not to change? Clearly, this can only be the case when the average rate of profit of the real economic system is equal to the average rate of profit of the Standard system. But, as we know, with unequal sectoral rates of profit, the average rate of profit will in general be different in the real economic system and in the Standard system. No contradiction will then be involved in the change in the average rate of profit when passing from one system to the other.

It seems, therefore, that Sinha's "formal argument" is based on the same misconception of his previous arguments: namely, on the assumption that the average rate of profit of the real economic system is always (that is, irrespective of prices) equal to 20%.[15]

There is, however, a particular case that needs to be considered. As we know, there are in general price systems which, without involving a uniform rate of profit (and hence without implying zero λ's), do assure that the average rate of profit of the real economic system is equal to the average rate of profit of its associated Standard system. As we saw in Section II, in the case of our real economic system these are the price systems that, together with the price system determined by equations [1a], satisfy our equation (3). Now, clearly, for *these* price systems we must expect the average rate of profit not to change in the passage from the real economic system to the Standard system. It would seem, then, that for this particular subset of the admissible prices the contradiction pointed out by Sinha actually exists. However, to see that this is not the case, it is sufficient to consider that, since equation (3) places a restriction on prices, it must place at the same time a restriction on the λ's: for the behaviour of such parameters is merely the reflection of the behaviour of prices. In particular, as we show in the Appendix, all strictly positive prices satisfying equation (3) and not satisfying at the same time equations [1a] — i.e., all prices assuring the same average rate of profit in our real economic system and in its associated Standard system without assuring a uniform rate of profit — are such as to imply a deviation from the average rate of profit in the wheat industry that, contrary to what is required by Sinha's argument, is opposite in sign to both the deviations in the iron and in the coal industries.

We may thus conclude that the contradiction envisaged by Sinha in his "formal argument" cannot present itself because, contrary to Sinha's own assertion concerning the absence of any "loss of generality" in his choice of the signs of the λ's, when the average rate of

profit is the same in the real economic system and in the Standard system, the signs of the λ's cannot possibly be such as to involve a change in the average rate of profit when passing from one system to the other.[16]

VIII. Concluding remarks

With his thesis according to which the condition of a uniform rate of profit would be a pure "logical necessity" in *Production of Commodities*, Sinha draws a marked distinction between Sraffa and the other classical economists. Indeed, Sinha has never denied that in Smith, Ricardo, and Marx the condition of a uniform rate of profit reflects the operation of competition among the owners of the means of production. But such a possibility is precisely what is ruled out by Sinha's thesis in Sraffa's case. Such a thesis, therefore, would have far-reaching implications both for the history of economic thought and for economic analysis. It is clear at the same time that, by its very nature, Sinha's thesis is independent of any strictly interpretative issue, in the sense that, aiming as it does at establishing a "logical necessity", it should be recognised, if true, by anyone who looks at Sraffa's work. It is for this reason that in this paper we have focused exclusively on the analytical arguments put forward by Sinha in support of his thesis.

The general aim pursued by Sinha with such arguments is easy to understand. In order to establish the "logical necessity" of a uniform rate of profit in *Production of Commodities*, Sinha would have to show that, in the context of Sraffa's work, unequal sectoral rates of profit involve a contradiction. More precisely, what Sinha would have to prove is the existence of a contradiction arising exclusively from the assumption of unequal sectoral rates of profit and from accepted premises of Sraffa's analysis (with the obvious exclusion of the condition of free competition). As we have seen, however, this he is unable to do. Indeed, whenever we come across a contradiction in his arguments, this is due to the fact that, along with a first proposition which can be correctly derived from Sraffa's analysis, Sinha more or less explicitly adds a second proposition of his own, which, besides being wrong, implies exactly the opposite of what is implied by the first proposition.

This emerges in a comparatively clear form in the two arguments Sinha has initially advanced in support of his thesis.

In the simplest of these arguments, a crucial role is ascribed to the equality between the maximum rate of expansion of the real economic system and that of its associated Standard system. It is such an equality that, in Sinha's mind, should provide the necessary condition

that would be contradicted by the assumption of unequal sectoral rates of profit. The fact is, however, that the maximum rate of expansion depends exclusively on the technical conditions of production, which the real economic system and the Standard system have in common. What Sinha has here is, then, a necessary equality; but what he would need in addition to this is the dependence of such an equality on the behaviour of the sectoral rates of profit. For, without the latter, he could not possibly establish the existence of a contradiction that would be implied by the assumption of unequal sectoral rates of profit. It is for this reason that Sinha explicitly adds to his argument the proposition stating that the maximum rate of expansion will be the same in the real economic system and in the Standard system "if and only if the sectoral rates of profit are equal in both systems". As a result, Sinha's argument ultimately relies in this case on the idea according to which the maximum rate of expansion of the real economic system is, and at the same time is not, independent of prices.

A similar situation arises in connection with Sinha's other initial argument, where, as in Sinha's more recent arguments, the comparison between the real economic system and the Standard system is carried out in terms of their average rate of profit. Unlike the maximum rate of expansion, the average rate of profit of the real economic system will generally change with changes in prices, while this does not happen to the average rate of profit of the Standard system. In this case, then, Sinha is correct in claiming that different sectoral rates of profit generally imply a different average rate of profit in the real economic system and in the Standard system. The equality that Sinha sets at the centre of this argument does in other terms depend on the behaviour of the sectoral rates of profit as he needs it to do. But there is no reason why such an equality should be a necessary equality. It is, then, in order to establish such a necessity — and hence the existence of a contradiction involved in the assumption of unequal sectoral rates of profit — that Sinha has to fall back to what is probably the most perplexing aspect of his general position: namely, the idea according to which, as is the case with the average rate of profit of the Standard system, the average rate of profit of the real economic system too would be independent of prices. Like the argument couched in terms of the maximum rate of expansion, this argument too can therefore be shown to be self-contradicting, as it is based on the idea according to which the average rate of profit of the real economic system depends, and at the same time does not depend, on prices.

At first sight, matters appear less straightforward in the case of Sinha's "formal argu-

ment". In this case, a crucial role is ascribed to the deviations of the sectoral rates of profit from the average rate — Sinha's λ's. In particular, in order to establish the contradictory nature of the assumption of unequal sectoral rates of profit, Sinha argues that, "without loss of generality", it would always be possible to choose the signs of the λ's — and hence the way in which the sectoral rates of profit can be found above and below the average rate — in such a way that the change in the proportions between the industries involved in the passage from the real economic system to the Standard system leads to a change in the average rate of profit. Now, it is clear in the first place that such a possibility would involve a contradiction only if, at the same time, the average rate of profit were the same in the real economic system and in the Standard system; and, as we know, it is true that, in addition to the price system implying a uniform rate of profit, there is generally a particular subset of the admissible prices for which the average rate of profit is the same in those two systems without this implying a uniform rate of profit. The fact is, however, that the same restriction on prices that ensures this result places a restriction on the possible combinations of the signs of the deviations of the sectoral rates of profit. More in particular, for prices comprised in the above-mentioned subset, the average rate of profit is always the same in the real economic system and in the Standard system, and the only possible combinations of such signs are those that, in the passage from one system to the other, do not imply a change in the average rate of profit. By contrast, for all other prices associated with unequal sectoral rates of profit, the average rate of profit is never the same in the real economic system and in the Standard system, and the only possible combinations of the signs of the λ's are those that, in the passage from one system to the other, imply a change in the average rate of profit.

In claiming that, when the signs of the λ's can be chosen in such a way as to imply a change in the average rate of profit, such a change would lead to a contradiction, Sinha is thus claiming that the properties that apply to two distinct subsets of the admissible prices hold at one and the same time. What he is claiming is, in fact, that the average rate of profit of the real economic system changes, and at the same time does not change, with the *same* changes in prices.

Naturally, such a contradiction in Sinha's position calls in turn for an explanation; and this, as we have suggested, should probably be sought in Sinha's belief in a "physical rate of profit" of the real economic system which we would be unable to see distinctly because of the "optical illusion" produced by prices. As we have seen, such an idea is entirely unwarranted, since what Sinha refers to as a "physical rate of profit" is only the ratio between two

different composite commodities. It does not seem unlikely, however, that such an idea underlies both Sinha's belief in an average rate of profit of the real economic system that would be a "physical property" of that system (and that would thus be such as not to change with changes in prices) *and* his insistence on the condition of a uniform rate of profit as the condition that, by making the average rate of profit equal in the real economic system and in the Standard system, would allow us to see the "physical nature" of such a rate.

Be that as it may, the purpose we have set for the present paper is that of assessing the validity of the analytical arguments Sinha has put forward in order to establish the "logical necessity" of a uniform rate of profit. And, in this respect, we can safely say that Sinha has not proved his thesis, so that we are left with no reason to believe that the condition of a uniform rate of profit has a different role in Sraffa's *Production of Commodities* from the one that such a condition has in the works of the classical economists and Marx.

References

Aspromourgos, T. (2012), Review of 'Ajit Sinha, *Theories of Value from Adam Smith to Piero Sraffa*', *European Journal of the History of Economic Thought*, 19(3): 490-495.

Carter, S. (2018), On the Hijacking of the Intellectual and Archival Legacy of Piero Sraffa: A Comment on Ajit Sinha (2016) – *A Revolution in Economic Theory: The Economics of Piero Sraffa*, *Bulletin of Political Economy*, 12(1-2): 3-34.

Dupertuis, M.S. and Sinha, A. (2009), A Sraffian critique of the classical notion of centre of gravitation, *Cambridge Journal of Economics*, 33(6): 1065-1087.

Fratini, S.M. (2012), Review of 'Ajit Sinha, *Theories of Value from Adam Smith to Piero Sraffa*', *Contributions to Political Economy*, 30(1): 102-105.

Kurz, H.D. (2012), Don't Treat Too Ill My Piero! Interpreting Sraffa's Papers, *Cambridge Journal of Economics*, 36(6): 1535-1569.

Levrero, E.S. (2012), Review of *Theories of Value from Adam Smith to Piero Sraffa*, *Review of Political Economy*, 24(3): 527-535.

Levrero, E.S. (2019), On Sinha's View of Sraffa's Revolution in Economic Theory: A Review Essay, *Review of Political Economy*, 31(1): 115-123.

Levrero, E.S. (2020), On Sraffa's Prices: A Rejoinder to Sinha, *Review of Political Economy*, 32(3): 450-458.

Pasinetti, L.L. (1977), *Lectures in the Theory of Production*, London: Macmillan.

Reati, A. (2012), Review of *Theories of Value from Adam Smith to Piero Sraffa*, *Review of Radical Political Economics*, 44(3): 395-400.

Reati, A. (2014), A Note on Some Misunderstanding of Sraffa's System, *Review of Radical Political Economics*, 46(3): 402-405.

Roncaglia, A. (2017), A. Sinha: A Revolution in Economic Theory: The Economics of Piero Sraffa, *Contributions to Political Economy*, 36(1): 139-144.

Schefold, B. (2021), Sraffa's Protoeconomics—A Comment on Ajit Sinha's 'Interpreting the Nature of Sraffa's Equations: A Critique of Garegnani's Interpretation', in A. Sinha (ed.), *A Reflection on Sraffa's Revolution in Economic Theory,* Cham: Palgrave Macmillan.

Sinha, A. (2007), Sraffa and the Assumption of Constant Returns to Scale: A Critique of Samuelson and Etula, *Contributions to Political Economy*, 26(1): 61-70.

Sinha, A. (2009), Sraffa and the Later Wittgenstein, *Contributions to Political Economy*, 28(1): 47-69.

Sinha, A. (2010), *Theories of Value from Adam Smith to Piero Sraffa*, London: Routledge.

Sinha, A. (2012), Listen to Sraffa's Silences: A New Interpretation of *Production of Commodities*, *Cambridge Journal of Economics*, 36(6): 1323-1339.

Sinha, A. (2014), A New Perspective on Sraffa, in R. Bellofiore and S. Carter (eds), *Towards a New Understanding of Sraffa. Insights from Archival Research,* Basingstoke: Palgrave Macmillan.

Sinha, A. (2016), *A Revolution in Economic Theory: The Economics of Piero Sraffa*, Cham: Palgrave Macmillan.

Sinha, A. (2018), A Response to My Critics, *STOREP Working Papers* 1-2018. Reprinted as an Afterword in *Theories of Value from Adam Smith to Piero Sraffa*, 2nd ed., 2018, London: Routledge.

Sinha, A. (2019), *Essays on Theories of Value in the Classical Tradition*, Cham: Palgrave Macmillan.

Sinha, A. (2021), Interpreting the Nature of Sraffa's Equations: A Critique of Garegnani's Interpretation, in A. Sinha (ed.), *A Reflection on Sraffa's Revolution in Economic Theory,* Cham: Palgrave Macmillan.

Sinha, A. and Dupertuis, M.S. (2009a), Sraffa's System: Equal Rate of Profits and the Notion of Center of Gravitation, *Journal of Economic Behavior & Organization*, 71(2): 495-501.

Sinha, A. and Dupertuis, M.S. (2009b), Sraffa and the Question of Equilibrium, *Cahiers d'Économie Politique*, 56: 91-100.

Smith, M. (2020), Review of 'Ajit Sinha, *Essays on Theories of Value in the Classical Tradition'*, *Contributions to Political Economy*, 39(1): 99-103.

Sraffa, P. (1960), *Production of Commodities by Means of Commodities. Prelude to a Critique of Economic Theory*, Cambridge: Cambridge University Press.

Notes

1 Helpful comments by Antonella Palumbo and Roberto Ciccone, as well as a number of insightful discussions with the latter, are gratefully acknowledged.

2 On this point, see in particular Levrero (2020), which offers a remarkable account of Sraffa's views on the role of competition in economic analysis.

3 Another issue raised by Sinha's contributions that we shall not consider in this paper concerns his use of Sraffa's unpublished papers. On this point, see Carter (2018).

4 An exception can be found in Reati (2012 and 2014), on which see note 12 below.

5 The relevant passages are reproduced both in Sinha and Dupertuis (2009b, p. 94) and in Sinha (2010, p. 291).

6 For Sinha's formulation of this proposition, see Section III.

7 On this point, see note 8.

8 It may be convenient to stress that what we have just said concerning the physical determination of the average rate of profit of the Standard system is what happens when, in building such a system, subsistence wages are included along with the means of production. When, instead, the Standard system is built starting exclusively from the means of production, what by means of that system can be determined without knowing prices is the *maximum* rate of profit, namely the limit that cannot be exceeded by the rate of profit when this is uniformly distributed across the industries of the system. In this case, the average rate of profit of the Standard system can be determined independently of prices only (a) by assuming zero wages (so that the average and the maximum rates of profit coincide), or (b) by using the inverse relation between the rate of profit and the wage rate. However, as has been rightly noted, respectively, by Reati (2012) and Roncaglia (2017), the assumption of zero wages is clearly untenable, while, in order to take wages as the independent variable in the inverse relation holding in the Standard system between the rate of profit and the wage rate, wages must be assumed to consist of Standard commodity; and, as we have seen, this is a composite commodity consisting both of consumption goods (in proportions which do not directly reflect workers' consumption habits or choices) and of means of production. On the other hand, these problems do not appear if, in building the Standard system, we include subsistence wages among the means of production: and there can be no doubt that, at least as long as what has to be established is the nature of the condition of a uniform rate of profit, such a case is not irrelevant for discussing *Production of Commodities*.

9 Clearly, a linear relation between two relative prices such as our equation (3) is what one obtains in the case of a three-basic-industry economic system. No such simple characterisation of the set of price systems compatible with the same average rate of profit in the real economic system and in the Standard system will generally be found in more complex cases. Incidentally, it is worth noting that, for an economic system producing two commodities, such set includes a single relative price, which means that the average rate of profit is the same in the real economic system and in the Standard system only when the rate of profit is uniform.

10 The criterion for recognising "equivalent systems" is explicitly stated in Dupertuis and Sinha (2009, p. 1072), within a critical discussion of the process of price gravitation. Three points should be made in this regard. (a) As we have said, the Standard system that we find in Sraffa is an *imaginary* system that allows us to determine the maximum rate of profit of the real economic system — or, under our assumption concerning wages, its uniform rate of profit — without knowing prices. The use of the same technical coefficients in the Standard system and in the real economic system does not, then, follow from any assumption on returns to scale: namely, on what would happen to the technical coefficients of production if the proportions between the various industries of the real economic system were *actually* to change. As we shall see in the next Section, Sin-

ha's use of the notion of "equivalent" systems is not always compatible with the absence of such an assumption. For this reason, we have set out Sinha's criterion for the equivalence between two production systems without explicitly stating the reason why the same production coefficients apply to such systems. (b) Sinha's criterion cannot be used without modifications beyond the case of basic-good systems. Indeed, according to such a criterion a real economic system and its associated Standard system would not be "equivalent" if the former system contained non-basic products. It seems legitimate, therefore, to extend the criterion by saying that, according to Sinha, any system that is obtained by a reallocation of the total amount of labour employed in the real economic system *to its basic industries* and that leaves the production coefficients unchanged is to be considered an "equivalent" system to such a real economic system. (c) A final remark must be made concerning Sinha's requirement of an unchanged amount of labour. This seems to be the result of Sinha's initial interest in the issue of price gravitation, as such an assumption is frequently used to separate questions relating to the composition of output from questions concerning its level. Indeed, the assumption does not seem to play any analytical role in Sinha's arguments aimed at proving the "logical necessity" of a uniform rate of profit. As noted above, however, it has the advantage that it eliminates any consideration regarding the scale in the determination of the Standard system associated to a given real economic system, thus making that system unique.

11 It should be noted that this is in fact not the case with respect to Sraffa's analysis. The reason is that, in order to be able to speak about the expansion of a given economic system, we would need to know how its yearly productive operations change as the produced quantity of the different commodities changes. But, as is very well known and as Sinha himself often points out (see, e.g., Sinha 2007 and 2021), Sraffa makes it particularly clear that the theses presented in his work involve "no changes in output" (Sraffa 1960, p. v) and hence do not depend on such knowledge.

12 The criticism we have raised in this Section against Sinha's argument based on the maximum rate of expansion recalls Reati's (2012 and 2014) criticism of the argument we have discussed in the previous two Sections. This is because Reati (misled by Sinha's assumption of null wages) interprets the variable R, which as we have seen is at the heart of Sinha's argument, not as the average rate of profit of the real economic system, but as its maximum rate of profit, which (like the maximum rate of expansion) depends exclusively on technical circumstances and is therefore equal in any two "equivalent systems" independently of the behaviour of the sectoral rates of profit.

13 The passage is taken from Sinha and Dupertuis (2009a, pp. 496-497), but also appears in Sinha (2010, p. 290) and, with minor changes, in Sinha (2014, p. 86).

14 It should be noted that, in drawing this conclusion, Sinha seems to overlook that, in the determination of the average rate of profit, if the absolute size of two industries changes, the relative weight of the rate of profit of a third industry whose absolute size is kept constant will change as well. This, however, can be shown not to be relevant for the validity his argument.

15 Indeed, once Sinha's views concerning the independence of the average rate of profit of the real economic system from prices are known, the presence of such views can be easily recognised in his "formal argument" by looking at the particular choice of notation. Indeed, throughout the presentation of such an argument, Sinha uses the same symbol R to denote the average rate of profit of the real economic system whether or not the rate of profit is uniform, whereas he uses two different symbols (p and p') to denote prices in the two different situations. The reason offered for such a choice is that "prices would change [...] if r's [...] are different from R" (Sinha 2016, p. 223), the implication clearly being that no change in the average rate of profit is expected by Sinha under the same circumstances.

16 Sinha's subsequent presentations of his "formal argument" add nothing of substance. In Sinha (2018), the economic system we have seen so far is abandoned in favour of an extremely simple two-industry system, and the argument is reformulated accordingly. In particular, here Sinha's point is that the conditions that establish that the sectoral rates of profit are expressed in terms of

deviations from the same average rate in both the real economic system and the Standard system cannot be simultaneously satisfied with non-zero deviations. But, contrary to what Sinha seems to believe, this has nothing to do with the necessity of a uniform rate of profit. The fact is more simply that, as we already mentioned (see note 9), when there are only two commodities there are no price systems other than the one assuring a uniform rate of profit which give the same average rate of profit in the real economic system and in the Standard system. In Sinha (2021), on the other hand, we come back to our three-industry real economic system. The only real difference to be noticed with respect to 2016's presentation is that here Sinha makes a rather explicit use of what we have called in Section V his "instrumental" assumption of a uniform rate of profit in order to determine the average rate of profit of the real economic system, a procedure that, as we have seen in that Section, can only be interpreted either as following from, or as being the intended proof of, the idea of an average rate of profit that would be given in the real economic system independently of prices.

Appendix

The purpose of this appendix is to prove the impossibility of a scenario such as the one envisaged in Sinha's "formal argument" in support of the "logical necessity" of a uniform rate of profit. As we have seen in Section VII, such an argument (Sinha 2016, pp. 222-225) essentially rests on the possibility of assuming *at the same time* (a) that the average rate of profit is the same in the real economic system and in its associated Standard system and (b) that the signs of the deviations of the sectoral rates of profit from the average rate are such that we must expect the average rate of profit to change when passing from one system to the other. It is, indeed, to avoid this contradiction that according to Sinha the deviations of the sectoral rates of profit "must take zero values" (p. 224). We shall prove that the two assumptions set out by Sinha are mutually inconsistent, i.e., that none of the price systems assuring the equality between the average rate of profit in the real economic system and in the Standard system implies signs of the deviations of the sectoral rates of profit that lead to a change in the average rate in the passage from one system to the other.

Let us consider an economic system producing three basic commodities x, y, and z, with q_x, q_y, and q_z being the multipliers capable of transforming the given system into its associated Standard system. In order to discuss the possibility of a scenario like the one postulated by Sinha, we need to take into consideration a Standard system in which one of the three commodities is produced in the same quantity as in the original system. There will thus be only two industries whose size must change to obtain our Standard system, while the third industry's multiplier will be equal to one. Clearly, we can find three Standard systems, or rather three alternative dimensions of the Standard system, which can serve our purposes. The one considered by Sinha is that in which, of the two industries whose size has to be changed, one must be expanded and the other contracted. Sinha assumes that the industry whose size increases has a higher rate of profit than the average rate, while the opposite is true for the industry whose size decreases. That being so, Sinha concludes that the average rate of profit that can be calculated for the Standard system must be necessarily higher than the one that can be calculated for the real economic system — or, more generally, that the average rate of profit will have to change in the passage from one system to the other.

Central to Sinha's argument is, therefore, the particular combination of the two non-unitary multipliers (q_x and q_y, if we assume $q_z = 1$) and the corresponding deviations from the average rate of profit (λ_x and λ_y). In particular, the combinations needed by Sinha are

the ones that assure that, in the passage from the real economic system to the Standard system, industries *x* and *y* exert their influence on the average rate of profit *in the same direction*. This can occur in the following cases: first, when the two industries display deviations from the average rate of profit of the same sign and, in order to obtain the Standard system, either both must shrink or both must expand (i.e., when $\lambda_x \lambda_y > 0$ and $(1 - q_x)(1 - q_y) > 0$); second, when the two industries display deviations from the average rate of profit of different sign and, in order to obtain the Standard system, one must shrink and the other must expand (i.e., when $\lambda_x \lambda_y < 0$ and $(1 - q_x)(1 - q_y) < 0$).

Now, it is true that by choosing both the system of relative prices and the size of the Standard system appropriately, it is in general possible to obtain one of the above cases. In order to do so, however, it will be necessary to choose price systems that generate in the real economic system a *different* average rate of profit from the one obtained in the Standard system. The change in the average rate of profit occurring in the passage from one system to the other will thus involve no contradiction.[1] When, on the contrary, the relative prices, and hence the deviations of the sectoral rates from the average, are such as to yield the same average rate of profit in the real economic system and in the Standard system, neither of the two cases can be expected to arise.

To show that this is the case, let us first define the average rate of profit of the real system, R, and the average rate of profit of the Standard system, R^*, by means of the following equations:

$$R = (r_x \mathrm{K}_x + r_y \mathrm{K}_y + r_z \mathrm{K}_z)/(\mathrm{K}_x + \mathrm{K}_y + \mathrm{K}_z) \tag{A1}$$

$$R^* = (r_x q_x \mathrm{K}_x + r_y q_y \mathrm{K}_y + r_z q_z \mathrm{K}_z)/(q_x \mathrm{K}_x + q_y \mathrm{K}_y + q_z \mathrm{K}_z) \tag{A2}$$

where K_j denotes the value of the capital advanced in the real system in industry j expressed in terms of some numeraire. From equations (A1) and (A2) we can derive:

$$(R - r_x)\mathrm{K}_x + (R - r_y)\mathrm{K}_y + (R - r_z)\mathrm{K}_z = 0 \tag{A3}$$

$$(R^* - r_x) q_x \mathrm{K}_x + (R^* - r_y) q_y \mathrm{K}_y + (R^* - r_z) q_z \mathrm{K}_z = 0 \tag{A4}$$

Let us now assume that the average rate of profit of the real system is equal to the average rate of profit of the Standard system, i.e., that $R = R^*$. This is tantamount to imposing a restriction on prices such as the one implied by our equation (3). We can however express that same restriction in terms of a condition involving the λ's. To this end, it suffices to put $R - r_j = R^* - r_j = \lambda_j$ in equations (A3) and (A4) to get:

$$\lambda_x \mathrm{K}_x + \lambda_y \mathrm{K}_y + \lambda_z \mathrm{K}_z = \lambda_x q_x \mathrm{K}_x + \lambda_y q_y \mathrm{K}_y + \lambda_z q_z \mathrm{K}_z. \tag{A5}$$

Since we are interested in the Standard system in which commodity z is produced in the same quantity as in the real system, we can set $q_z = 1$ and obtain the following equation:

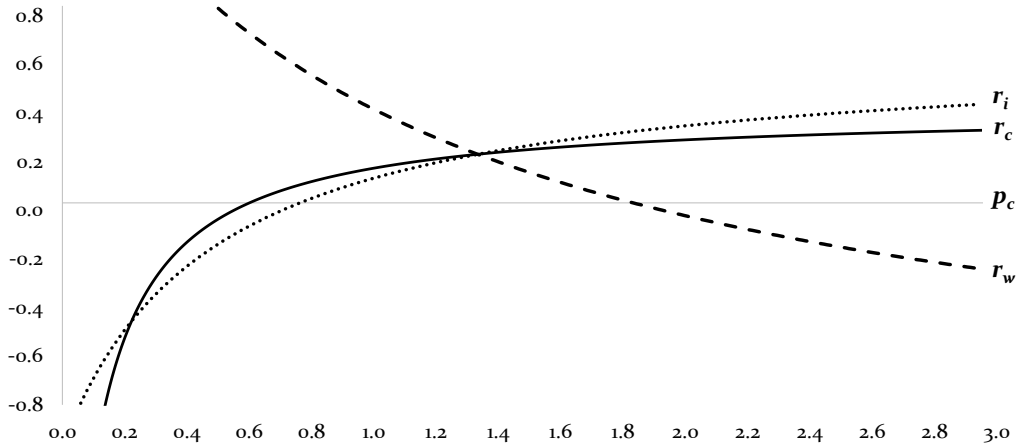
$$\lambda_x = -[(1 - q_y)/(1 - q_x)](K_y/K_x)\,\lambda_y. \tag{A6}$$

For the economic system producing iron, coal, and wheat to which Sinha (2016) refers, equation (A6) yields:

$$\lambda_i = [(33p_c + 16)/(78p_c + 6)]\,\lambda_c, \tag{A7}$$

which implies that, for positive prices, when $R = R^*$ the deviations λ_i and λ_c are always concordant, as shown in figure 1.[2]

Figure 1 – Behaviour of the sectoral rates of profit for relative prices consistent with an average rate of profit equal to 20%.



The figure shows the combinations of sectoral rates of profit consistent with relative prices satisfying equation (3), or, equivalently, with $R = 20\%$. The rate of profit is uniform, i.e., $r_i = r_c = r_w = R$, for $p_c = 4/3$. If $p_c > 4/3$, then $r_i > R$, $r_c > R$, and $r_w < R$; whereas if $0 < p_c < 4/3$, then $r_i < R$, $r_c < R$, and $r_w > R$. In both cases, $\lambda_i\,\lambda_c > 0$, as prescribed by equation (A7).

In general, equation (A6) shows that, if prices are all positive (so that $K_y/K_x > 0$), when $(1 - q_x)$ and $(1 - q_y)$ are concordant, for R to be equal to R^*, λ_x and λ_y must be discordant, while if $(1 - q_x)$ and $(1 - q_y)$ are discordant, then λ_x and λ_y must be concordant. In other words, when, with unequal sectoral rates of profit, the average rate of profit is the same in the real economic system and in the Standard system, then, if industries x and y are both smaller or both larger in the Standard system than in the real system, they necessarily have profit rates one above and the other below the average (i.e., if $(1 - q_x)(1 - q_y) > 0$, then $\lambda_x\,\lambda_y < 0$), while, if one of the two industries is larger and the other smaller, they exhibit deviations from the average of the same sign (i.e., if $(1 - q_x)(1 - q_y) < 0$, then $\lambda_x\,\lambda_y > 0$). Hence, such a scenario as the one envisaged by Sinha, in which $(1 - q_x)(1 - q_y) < 0$ and $\lambda_x\,\lambda_y < 0$, cannot possibly arise.

Notes for the Appendix

1 For the economic system referred to by Sinha, we can give an example that reproduces the exact case he considers, namely $\lambda_i > 0$, $\lambda_c < 0$, and $\lambda_w < 0$ with $q_i > 1$, $q_c < 1$, and $q_w = 1$. When $p_i = 13/3$ and $p_c = 17/12$, the iron industry has an above-average profit rate, while the coal and wheat industries have below-average profit rates ($r_i = 26\%$, $r_c = 17\%$, and $r_w = 12\%$). In this case it is true that the average rate of profit of the Standard system is higher than that of the real system, but the latter is 19%, i.e., it is lower than what would be the result of a uniform rate of profit.

2 The deviations λ_i and λ_c will have opposite signs only for $-16/33 < p_c < -1/13$. As irrelevant as such a price range is, let us note that the contradiction figured by Sinha by no means occurs here either. The invariance of the average rate of profit is preserved despite the fact that in the Standard system, with respect to the real system, the industry with an above-average rate of profit is larger and the industry with a below-average rate of profit is smaller. For, while it is true that for R^* the weight of the iron industry's profit rate is greater than for *R in absolute terms*, since this weight has a negative sign, the contribution of the iron industry to the average rate of profit is actually smaller in the Standard system than in the real system.