Endogenous Switching of Volatility Regimes: Rational Expectations and Behavioral Sunspots

Gaetano Gaballo

University of Siena

March 24, 2009

Facts: changes in inflation volatility regimes

Figure: US inflation (percentage change of Production Prices Index) time series of last fifty years. Grey bars denote recession periods. The picture suggests different volatility regimes with no strict correlation between fluctuation amplitude and growth cycles.

This paper...

- ...is not an empirical study but a theoretical one.

This paper...

- ...is not an empirical study but a theoretical one.
- The task is to provide a stylized model on how *endogenous*, *unpredictable* and *persistent* switches in volatility regimes can arise mainly because agents fail to form expectations *independently*.

This paper...

- ...is not an empirical study but a theoretical one.
- The task is to provide a stylized model on how *endogenous, unpredictable* and *persistent* switches in volatility regimes can arise mainly because agents fail to form expectations *independently*.
- The literature on excess volatility of market asset returns is really huge (Timmermann 1993,1996, Adam, Marcet, Nicolini 2008, Brock and Hommes 1997,1998, Allen, Morris and Shin 2006), whereas very little has been written on theoretical explanation of stochastic volatility of inflation (Evans and Branch 2007). Those models rely on some mechanism that either is exogenously imposed at an aggregate level or persistently alters the volatility regime of the dynamics.

This paper...

- ...is not an empirical study but a theoretical one.
- The task is to provide a stylized model on how *endogenous, unpredictable* and *persistent* switches in volatility regimes can arise mainly because agents fail to form expectations *independently*.
- The literature on excess volatility of market asset returns is really huge (Timmermann 1993,1996, Adam, Marcet, Nicolini 2008, Brock and Hommes 1997,1998, Allen, Morris and Shin 2006), whereas very little has been written on theoretical explanation of stochastic volatility of inflation (Evans and Branch 2007). Those models rely on some mechanism that either is exogenously imposed at an aggregate level or persistently alters the volatility regime of the dynamics.
- The most important features of the proposed model in front of previous literature is that: *i*) the model is able to reconcile REE behavior to volatility crises *ii*) moreover the extra noise possibly entering in the equilibrium solution is justified at a micro level

The starting point: along an expectationalist perspective.

- Pre-eminence of communication policy on monetary determinants. **Behavioral uncertainty** and the "forecasting the forecasts of others problem".

The starting point: along an expectationalist perspective.

- Pre-eminence of communication policy on monetary determinants. **Behavioral uncertainty** and the "forecasting the forecasts of others problem".
- Institutional mechanism for beliefs convergence: central bank act as **focal point (**Morris and Shin 2002**)**.

The starting point: along an expectationalist perspective.

- Pre-eminence of communication policy on monetary determinants. **Behavioral uncertainty** and the "forecasting the forecasts of others problem".
- Institutional mechanism for beliefs convergence: central bank act as **focal point (**Morris and Shin 2002**)**.
- This is an empirical fact!

The starting point: along an expectationalist perspective.

- Pre-eminence of communication policy on monetary determinants. **Behavioral uncertainty** and the "forecasting the forecasts of others problem".
- Institutional mechanism for beliefs convergence: central bank act as **focal point (**Morris and Shin 2002**)**.
- This is an empirical fact!
- Agents are not able to hold rational expectations *independently.*

The starting point: along an expectationalist perspective.

- Pre-eminence of communication policy on monetary determinants. **Behavioral uncertainty** and the "forecasting the forecasts of others problem".
- Institutional mechanism for beliefs convergence: central bank act as **focal point (**Morris and Shin 2002**)**.
- This is an empirical fact!
- Agents are not able to hold rational expectations *independently.*
- Gathering and processing information is really expensive (extreme economies of scale).

The starting point: along an expectationalist perspective.

- Pre-eminence of communication policy on monetary determinants. **Behavioral uncertainty** and the "forecasting the forecasts of others problem".
- Institutional mechanism for beliefs convergence: central bank act as **focal point (**Morris and Shin 2002**)**.
- This is an empirical fact!
- Agents are not able to hold rational expectations *independently.*
- Gathering and processing information is really expensive (extreme economies of scale).
- **Adaptive learning** (Marcet and Sargent 1989, Evans and Honkapohja 2001) approach answers the need of a more reasonable and dynamic theory of expectation formation. The typical focus is on exogenous uncertainty.

Merging and innovating the two basic ideas

- **Two institutional forecasters:** rating agencies, market leaders, fiscal authorities generally influence private sector expectations as well, and sometimes more, than the central bank. This implies arising of strategic motives in expectations formation. (Forecasting the forecasts of others problem!)

Merging and innovating the two basic ideas

- **Two institutional forecasters:** rating agencies, market leaders, fiscal authorities generally influence private sector expectations as well, and sometimes more, than the central bank. This implies arising of strategic motives in expectations formation. (Forecasting the forecasts of others problem!)
- **Learning about fundamentals and others rationality:** each institutional forecasters uses adaptive learning not only to learn about the fundamentals (as standard in literature) but also to assess the rationality of the other institutional forecaster's expectations (Gaballo 2009).

Merging and innovating the two basic ideas

- **Two institutional forecasters:** rating agencies, market leaders, fiscal authorities generally influence private sector expectations as well, and sometimes more, than the central bank. This implies arising of strategic motives in expectations formation. (Forecasting the forecasts of others problem!)
- **Learning about fundamentals and others rationality:** each institutional forecasters uses adaptive learning not only to learn about the fundamentals (as standard in literature) but also to assess the rationality of the other institutional forecaster's expectations (Gaballo 2009).
- **Non-neutrality of the information transmission channel to the private sector:** the problem faced by institutional forecasters is not their mere expectations coordination problem because the non-neutrality of the information transmission channel form them to the ocean of agents forming the private sector possibly alters the feedback mechanism (truly macroeconomic flavor).

In sum, I'm going to consider an economy so shaped

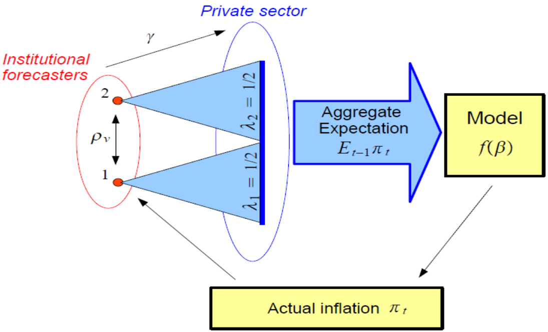

Figure: Information diffusion in the economy. Institutional forecasters 1 and 2 analyse data as they become available in time and produce statistically optimal forecasts. Institutional forecasters' expectations polarize evenly private sector expectations. The latter determine, jointly with other exogenous determinants, the actual inflation.

Model

- It is a microfounded Lucas-type monetary model (Evans and Branch 2007) yielding the following reduced form

$$\pi_t = \alpha\omega_{t-1} + \beta\pi_t^e + \nu_t \quad \text{with} \quad \beta^* \in (0,1)$$

whose unique REE is

$$\overline{\pi}_t = \frac{\alpha}{1-\beta}\omega_{t-1} + \nu_t.$$

Model

- It is a microfounded Lucas-type monetary model (Evans and Branch 2007) yielding the following reduced form

$$\pi_t = \alpha\omega_{t-1} + \beta\pi_t^e + \nu_t \quad \text{with} \quad \beta^* \in (0,1)$$

whose unique REE is

$$\overline{\pi}_t = \frac{\alpha}{1-\beta}\omega_{t-1} + \nu_t.$$

- Main hypothesis are: i) money in the utility model ii) money demand is inelastic to interest rate (Walsh 2003) and iii) a non negligible fraction of firm set price a period before.

Behavioral Uncertainty and Structural Heterogeneity

- Behavioral uncertainty

$$\left(E^{i}_{t-1}E^{j}_{t-1}\pi_t - E^{j}_{t-1}\pi_t\right) \equiv v_{i,t-1} \sim \Upsilon\left(0, \delta\right)$$

Behavioral Uncertainty and Structural Heterogeneity

- Behavioral uncertainty

$$\left(E^i_{t-1}E^j_{t-1}\pi_t - E^j_{t-1}\pi_t\right) \equiv v_{i,t-1} \sim \Upsilon\left(0, \delta\right)$$

- Institutional forecasters' expectations

$$E^i_{t-1}x_t \equiv E[x_t|\Omega^i_{t-1}], \ \ \forall i = 1, 2$$

Behavioral Uncertainty and Structural Heterogeneity

- Behavioral uncertainty

$$\left(E^i_{t-1}E^j_{t-1}\pi_t - E^j_{t-1}\pi_t\right) \equiv v_{i,t-1} \sim \Upsilon\left(0,\delta\right)$$

- Institutional forecasters' expectations

$$E^i_{t-1}x_t \equiv E[x_t|\Omega^i_{t-1}], \ \ \forall i = 1,2$$

- Private sector expectations

$$E^z_{t-1}\pi_t = E^{i_z}_{t-1}\pi_t + v_{z,t-1},$$

Behavioral Uncertainty and Structural Heterogeneity

- Behavioral uncertainty

$$\left(E^{i}_{t-1}E^{j}_{t-1}\pi_t - E^{j}_{t-1}\pi_t\right) \equiv v_{i,t-1} \sim \Upsilon\left(0,\delta\right)$$

- Institutional forecasters' expectations

$$E^{i}_{t-1}x_t \equiv E[x_t|\Omega^{i}_{t-1}], \ \ \forall i = 1,2$$

- Private sector expectations

$$E^{z}_{t-1}\pi_t = E^{i_z}_{t-1}\pi_t + v_{z,t-1},$$

- Aggregate expectation

$$E_{t-1}\pi_t = \int_{z\in Z} E^{z}_{t-1}\pi_t \ dz = \frac{1}{2}\sum_{i=1,2} E^{i}_{t-1}\pi_t + \int_{z\in Z} v_z \ dz$$

Institutional forecasters' expectations

- They learn about fundamental inflation (long-run component)

$$E^i_{t-1}\overline{\pi}_t = \mathbf{a}'_{i,t-1}\mathbf{z}_{t-1} \text{ with } \mathbf{z}'_{t-1} \equiv [1, \omega_{t-1}]$$

where $\mathbf{a}_{i,t-1}$ is updated according to a recursive OLS algorithm. Notice they soon converge to the same estimate $\overline{\pi}^e_t$!!!

Institutional forecasters' expectations

- They learn about fundamental inflation (long-run component)

$$E^i_{t-1}\overline{\pi}_t = \mathbf{a}'_{i,t-1}\mathbf{z}_{t-1} \quad \text{with} \quad \mathbf{z}'_{t-1} \equiv [1, \omega_{t-1}]$$

where $\mathbf{a}_{i,t-1}$ is updated according to a recursive OLS algorithm. Notice they soon converge to the same estimate $\overline{\pi}^e_t$!!!

- They learn about an idiosyncratic deviation from it possibly due to non-rationality of others' expectations (the idiosyncratic component)

$$E^1_{t-1}\left(\pi_t - \overline{\pi}^e_t\right) = b_{t-1}\ \phi^1_{t-1},$$

$$E^2_{t-1}\left(\pi_t - \overline{\pi}^e_t\right) = c_{t-1}\ \phi^2_{t-1},$$

$$\text{with } \phi^i_{t-1} = [E^j_{t-1}\pi_t + v_{i,t-1} - \overline{\pi}^e_t]'$$

where b_{t-1} and c_{t-1} is updated according to a recursive OLS algorithm.

And finally the actual law of motion

- Finally we can nest everything, so that we can specify actual inflation dynamics as

$$\pi_t = \alpha\omega_{t-1} + \beta\overline{\pi}_t^e + \frac{(1+\gamma)\beta}{2}\left(\frac{b(1+c)}{1-bc}v_{1,t-1} + \frac{c(b+1)}{1-bc}v_{2,t-1}\right) + v_t \ .$$

And finally the actual law of motion

- Finally we can nest everything, so that we can specify actual inflation dynamics as

$$\pi_t = \alpha\omega_{t-1} + \beta\overline{\pi}_t^e + \frac{(1+\gamma)\,\beta}{2}\left(\frac{b\,(1+c)}{1-bc}v_{1,t-1} + \frac{c(b+1)}{1-bc}v_{2,t-1}\right) + v_t \ .$$

- Notice that $\pi_t = \overline{\pi}_t^e$ if: $i)$ agents are not uncertain about others' behavior, that is $v_{1,t-1} = 0$ and $v_{2,t-1} = 0$, or $ii)$ agents hold the rational expectation, that is, $b_{t-1}^1 = 0$ and $c_{t-1} = 0$, or $iii)$ expectations have a zero impact on the actual course given $\beta^* = 0$.

And finally the actual law of motion

- Finally we can nest everything, so that we can specify actual inflation dynamics as

$$\pi_t = \alpha\omega_{t-1} + \beta\overline{\pi}_t^e + \frac{(1+\gamma)\,\beta}{2}\left(\frac{b\,(1+c)}{1-bc}v_{1,t-1} + \frac{c(b+1)}{1-bc}v_{2,t-1}\right) + v_t \ .$$

- Notice that $\pi_t = \overline{\pi}_t^e$ if: *i*) agents are not uncertain about others' behavior, that is $v_{1,t-1} = 0$ and $v_{2,t-1} = 0$, or *ii*) agents hold the rational expectation, that is, $b_{t-1}^1 = 0$ and $c_{t-1} = 0$, or *iii*) expectations have a zero impact on the actual course given $\beta^* = 0$.
- Otherwise inflation will exhibit endogenous excess volatility around the estimated inflation due by the stochastic term multiplied by $(1+\gamma)\,\beta$.

Equilibria: definition

- **Definition:** Equilibria obtain as fix points of the T map for $T_{\mathbf{a}}\left(\widehat{\mathbf{a}}\right) = \alpha$, $T_b\left(\widehat{b}, \widehat{c}\right) = \widehat{b}$ and $T_c\left(\widehat{b}, \widehat{c}\right) = \widehat{c}$.

Equilibria: definition

- **Definition:** Equilibria obtain as fix points of the T map for $T_{\mathbf{a}}(\widehat{\mathbf{a}}) = \alpha$, $T_b\left(\widehat{b}, \widehat{c}\right) = \widehat{b}$ and $T_c\left(\widehat{b}, \widehat{c}\right) = \widehat{c}$.
- T-map are obtained as solution of the following system

$$
\begin{aligned}
\mathbf{E}[\mathbf{z}_{t-1}\left(\pi_t - T_{\mathbf{a}}(\mathbf{a})\,\mathbf{z}_{t-1}\right)] &= 0, \\
\mathbf{E}[\phi^1_{t-1}\left(\pi_t - \overline{\pi}^e_t - T_b(b,c)\,\phi^1_{t-1}\right)] &= 0, \\
\mathbf{E}[\phi^2_{t-1}\left(\pi_t - \overline{\pi}^e_t - T_c(b,c)\,\phi^2_{t-1}\right)] &= 0.
\end{aligned}
$$

Equilibria: definition

- **Definition:** Equilibria obtain as fix points of the T map for $T_{\mathbf{a}}\left(\widehat{\mathbf{a}}\right) = \alpha$, $T_b\left(\widehat{b}, \widehat{c}\right) = \widehat{b}$ and $T_c\left(\widehat{b}, \widehat{c}\right) = \widehat{c}$.
- T-map are obtained as solution of the following system

$$
\begin{aligned}
\mathbf{E}[\mathbf{z}_{t-1}\left(\pi_t - T_{\mathbf{a}}\left(\mathbf{a}\right)\mathbf{z}_{t-1}\right)] &= 0, \\
\mathbf{E}[\phi_{t-1}^1\left(\pi_t - \overline{\pi}_t^e - T_b\left(b, c\right)\phi_{t-1}^1\right)] &= 0, \\
\mathbf{E}[\phi_{t-1}^2\left(\pi_t - \overline{\pi}_t^e - T_c\left(b, c\right)\phi_{t-1}^2\right)] &= 0.
\end{aligned}
$$

- T. map is a function giving the coefficients of the linear forecast of π_t yielding the smaller mean square error variance conditioned on the available information set (Marcet and Sargent 1989).

Equilibria: definition

- **Definition:** Equilibria obtain as fix points of the T map for $T_{\mathbf{a}}(\widehat{\mathbf{a}}) = \alpha$, $T_b\left(\widehat{b}, \widehat{c}\right) = \widehat{b}$ and $T_c\left(\widehat{b}, \widehat{c}\right) = \widehat{c}$.
- T-map are obtained as solution of the following system

$$\begin{aligned}
\mathbf{E}[\mathbf{z}_{t-1}\left(\pi_t - T_{\mathbf{a}}(\mathbf{a})\,\mathbf{z}_{t-1}\right)] &= 0, \\
\mathbf{E}[\phi^1_{t-1}\left(\pi_t - \overline{\pi}^e_t - T_b(b, c)\,\phi^1_{t-1}\right)] &= 0, \\
\mathbf{E}[\phi^2_{t-1}\left(\pi_t - \overline{\pi}^e_t - T_c(b, c)\,\phi^2_{t-1}\right)] &= 0.
\end{aligned}$$

- T. map is a function giving the coefficients of the linear forecast of π_t yielding the smaller mean square error variance conditioned on the available information set (Marcet and Sargent 1989).
- **Definition:** BSE are equilibria for which institutional forecasters' expectations exhibit interdependence, that is, BSE are equilibria $(\widehat{\mathbf{a}}, \widehat{b}, \widehat{c})$ such that $(\widehat{b}, \widehat{c}) \neq (0, 0)$.

Equilibria: T-map

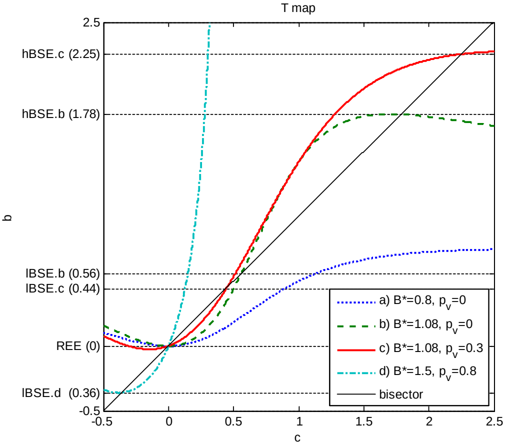

Figure: Tmap representation for different calibration. Equilibria are at intersection with Tmap with the bisector. For values of β^* bigger than one two BSEs arise besides REE.

Simulations: Benchmark case, real time

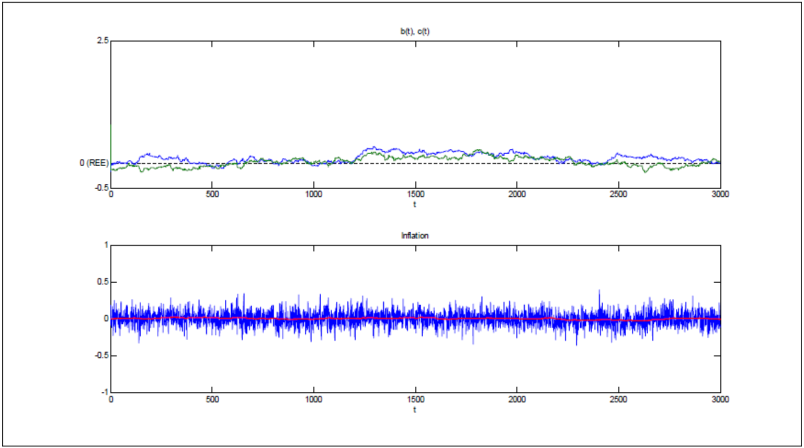

Figure: Benchmark case. Convergence to REE ($\beta = 0.8$, $\gamma = 0$, $\rho_v = 0$). Line a). in figure 3.

Simulations: High BSE arising I, real time

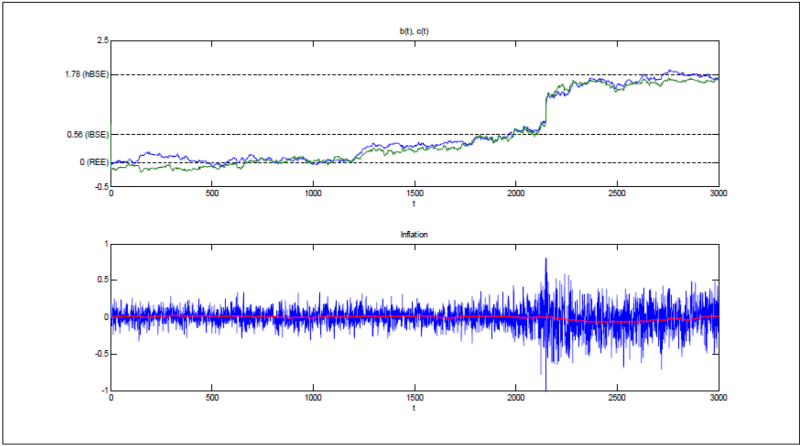

Figure: From REE to hBSE ($\beta = 0.8$, $\gamma = 0.28$, $\rho_v = 0$). Line b). in figure 3.

Simulations: High BSE arising II

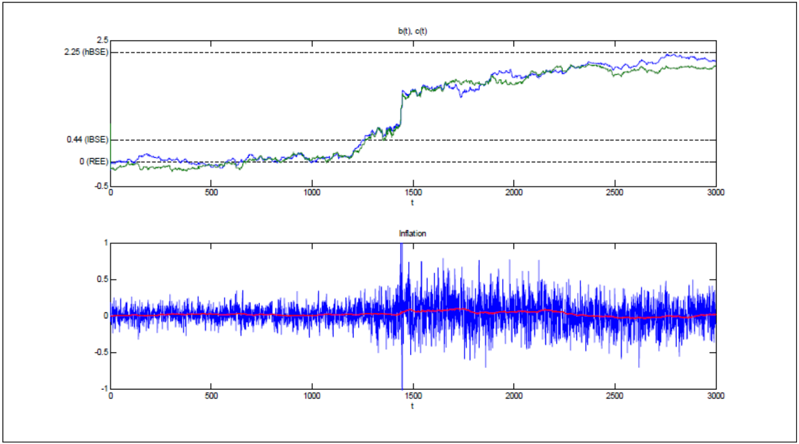

Figure: From REE to hBSE ($\beta = 0.8$, $\gamma = 0.28$, $\rho_v = 0.3$). Line c). in figure 3.

Simulations: from REE to high BSE and back

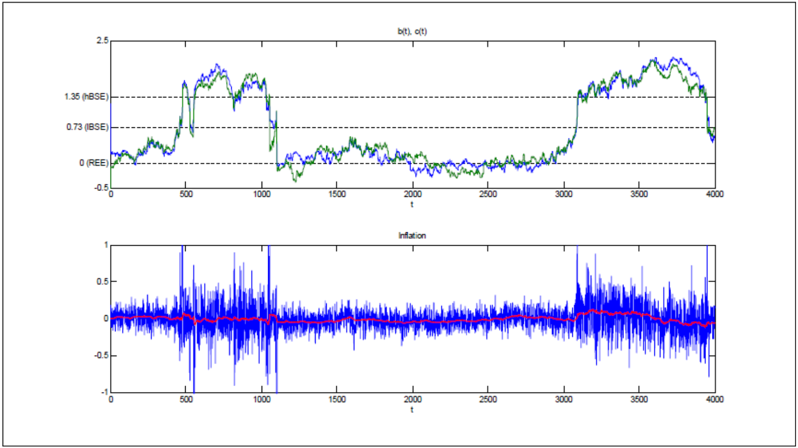

Figure: From REE to hBSE and back two times ($\beta = 0.8$, $\gamma = 0.21$, $\rho_v = 0.4$).

Conclusion

- Interactive learning generates convergence to the fundamental REE provided some condition on β and γ are met. By the way In a non trivial region of the coefficients space BSE emergence and disappearance are unpredictable;

Conclusion

- Interactive learning generates convergence to the fundamental REE provided some condition on β and γ are met. By the way In a non trivial region of the coefficients space BSE emergence and disappearance are unpredictable;
- Such convergence occurs in real time and without assuming any common knowledge property. This result is robust to correlation in observational errors;

Conclusion

- Interactive learning generates convergence to the fundamental REE provided some condition on β and γ are met. By the way In a non trivial region of the coefficients space BSE emergence and disappearance are unpredictable;
- Such convergence occurs in real time and without assuming any common knowledge property. This result is robust to correlation in observational errors;
- Existence of high BSE (high volatility regimes) is due to amplification of behavioral uncertainty effects generated through the information diffusion channel from PFs to the public;

Conclusion

- Interactive learning generates convergence to the fundamental REE provided some condition on β and γ are met. By the way In a non trivial region of the coefficients space BSE emergence and disappearance are unpredictable;
- Such convergence occurs in real time and without assuming any common knowledge property. This result is robust to correlation in observational errors;
- Existence of high BSE (high volatility regimes) is due to amplification of behavioral uncertainty effects generated through the information diffusion channel from PFs to the public;
- At least one BSE is learneable whenever BSEs exist. Differently from the most part of sunspots equilibria, agents' coordination arises endogenously. Behavioral uncertainty can produce persistent deviations from REE;

Conclusion

- Interactive learning generates convergence to the fundamental REE provided some condition on β and γ are met. By the way In a non trivial region of the coefficients space BSE emergence and disappearance are unpredictable;
- Such convergence occurs in real time and without assuming any common knowledge property. This result is robust to correlation in observational errors;
- Existence of high BSE (high volatility regimes) is due to amplification of behavioral uncertainty effects generated through the information diffusion channel from PFs to the public;
- At least one BSE is learneable whenever BSEs exist. Differently from the most part of sunspots equilibria, agents' coordination arises endogenously. Behavioral uncertainty can produce persistent deviations from REE;
- Endogenous, unpredictable and persistent switch from REE to high BSE and viceversa can arise without assuming any common knowledge property (Markov switching) or additional aggregate shock;

Thanks for your attention