The Financial Instability Hypothesis: a Stochastic Microfoundation Framework

Carl Chiarella and Corrado Di Guilmi

School of Finance and Economics - University of Technology, Sydney

The complexity of financial crisis in a long-period perspective: facts, theory and models

Siena

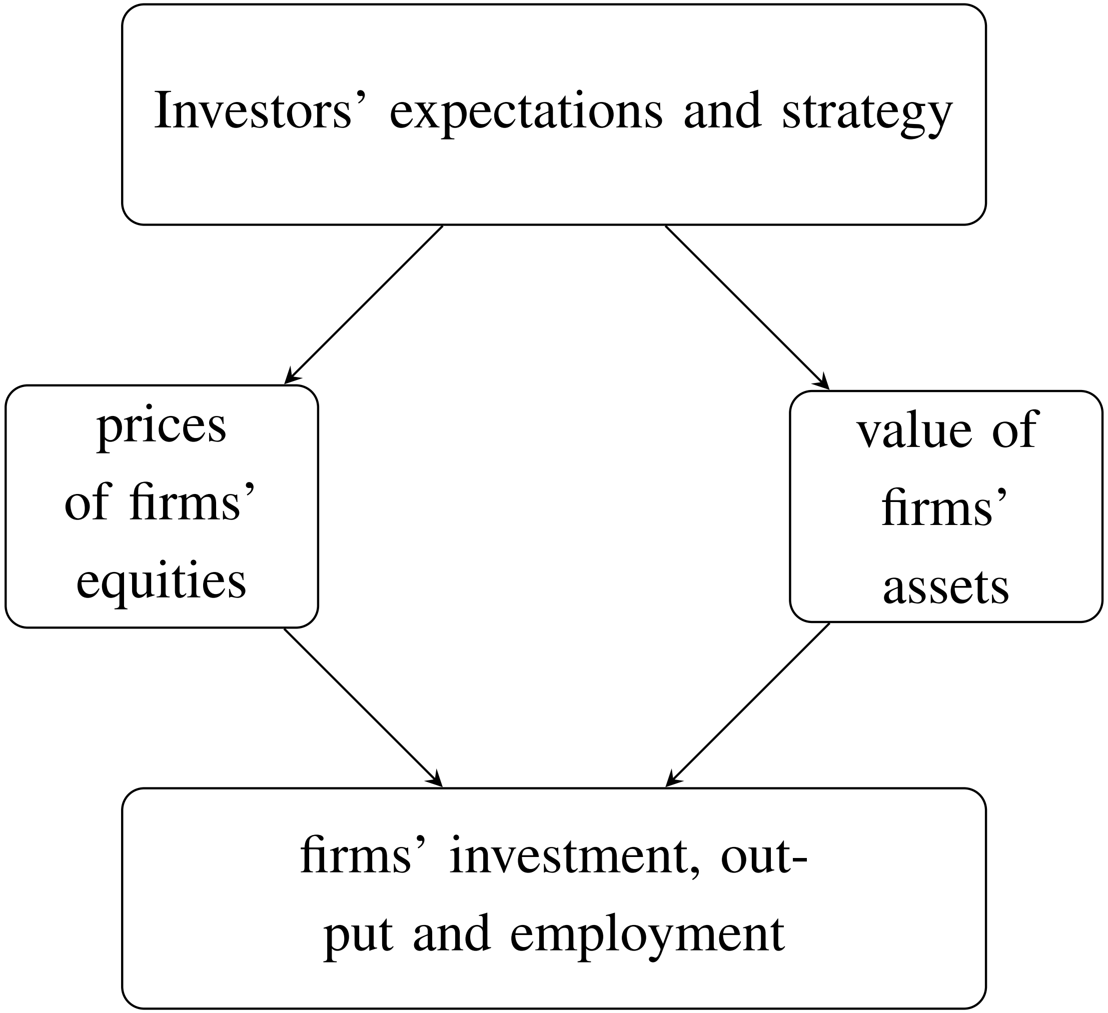
March 24, 2009

1 Introduction

1.1 Aims

- To consistently microfound the model of financial instability proposed by Minsky (1975) and Taylor and O'Connell (1985) in which variations in the price of firms assets drive investments, according to the mechanism first described by Keynes.
- To analyse the impact of fluctuations in financial markets on firms' micro-financial variables, and, through this, on the macroeconomy.

1.2 Outline of the model



- The market forecasts a rise (drop) in the demand for a good.

$\downarrow$

- Increase (decrease) in the market value of the *machines* that produce that good and a rise (drop) in the price of *shares* for the firms that sell them.

$\downarrow$

- At the aggregate level the economy may experience periods of growth, depression or fluctuations due solely to changes in the market mood and not to its actual productivity.

1.3 The main issues

- **Heterogeneous and interacting agents**: firms are different with respect to *financial structure* (Minsky 1982) and their situations are subject to *feedback effects* within the system:
  - a representative agent framework is not useful in this context;
- The analysis should be **dynamic** and, given the inherent uncertainty in the decision process, formulated in **probabilistic terms**;

$$\downarrow$$

The model adopts the stochastic dynamic aggregation framework proposed in Aoki (2006) and Di Guilmi (2008).

2 Hypothesis

2.1 Firms

- The number of firms is fixed and equal to N.
- Firms are classified into two groups according to their level of debt D:
  - state $z = 1$: **speculative firms**: $D(t) > 0$
  - state $z = 2$: **hedge firms**: $D(t) = 0$

- A firm decides on investment based on the the shadow-price of capital $P_k^j(t)$:

$$P_k^j(t) = \frac{(r(t) + \rho^j(t))P}{\iota(t)} \tag{1}$$

  where:

  - ρ^j is the expected difference of return to capital for the firm j with respect to the average level r;
  - ι is the interest rate and P is the final good price.

- Investment i is equal to:

$$\boxed{i^j(t) = aP_k^j(t)} \tag{2}$$

  with a as a parameter.

- Firms prefer to finance their investments first with retained earnings n and, then, with new equities e or debt D.
- The part of investment that is not covered with internal funds is financed by a fraction $\phi\ \iota(t)$ with equities and then the rest with debt, so that:

$$\boxed{\begin{aligned} de^j(t) &= \phi\ \iota(t)\left[P_k^j(t)i^j(t) - n(t)\right]dt \\ dD^j(t) &= \left[1 - \phi\ \iota(t)\right]\left[P_k^j(t)i^j(t) - n(t)\right]dt \end{aligned}} \tag{3}$$

- A firm fails if $D^j(t) > cK^j(t)$, with $c > 1$, and it is replaced by a new one.

- The balance sheet of a typical firm has the structure shown in table 1.

Assets	Liabilities
$\frac{(r+\rho)P}{\iota}k$	$P_e e$
	D (or n)

Table 1: Structure of a generic firm's balance sheet. Note that k is firm's capital.

- Asset prices P_{e1} and P_{e2} are subject to changes due to variations in investors' expectations and strategies.
- The path of capital accumulation in the economic system is then:

$$\boxed{\begin{aligned} \frac{d(P_k(t)K(t))}{dt} &= P_k(t)I(t) + \frac{dP_k(t)}{dt}K(t) \\ &= \frac{dP_e}{dt}E(t) + P_e\frac{dE(t)}{dt} + \frac{dD(t)}{dt} + \frac{dN(t)}{dt} \end{aligned}} \tag{4}$$

2.2 Investors

- Investors allocate their wealth W according to

$$\begin{cases} \epsilon_1(\iota, r + \rho_1)W = P_{e,1}E_1 \\ \epsilon_2(\iota, r + \rho_2)W = P_{e,2}E_2 \\ \beta(\iota, r + \rho_1)W = D \end{cases} \tag{5}$$

  with $\epsilon_1 + \epsilon_2 + \beta = 1$.

- The proportion of chartist in the market n^c is randomly drawn from a uniform distribution.

- The key variable for the allocation of wealth is ρ^j:
  - It reflects the investors' expectations on the different firms;
  - It determines the level of firms' investment;
  - In this way, it *measures the impact of financial market on the real sector*;
- Analytically
  - ρ_z^j are assumed to be functions of n^c:

$$\rho_1^j = \frac{n^c}{\tilde{\varpi}^j}\ , \qquad \rho_2^j = \frac{1 - n^c}{\tilde{\varpi}^j}$$

    where $\tilde{\varpi}^j$ is an idiosyncratic random variable;
  - The values ρ_1 and ρ_2 are computed as mean-field approximations.

- Consequently, the model is able to generate dynamics in two different ways:
  - an *agent based* approach with N different agents;
  - a *stochastic approximation*, with $\mathbf{2}$ different firms: one ”good” and one ”stressed”.

3 Stochastic dynamics

3.1 Transition probabilities

- The transition probabilities are indicated as
  - ζ: probability for a firm to move from state $\mathbf{2}$ to state $\mathbf{1}$;
  - ν: probability to move from state $\mathbf{1}$ to state $\mathbf{2}$.
- The transition rates are:
  - λ: probability of observing a transition from state $\mathbf{2}$ to state $\mathbf{1}$ in a unit of time;
  - μ: probability of observing a transition from state $\mathbf{1}$ to state $\mathbf{2}$ in a unit of time.

- The transition probabilities can be expressed as functions of n^c:

$$\begin{aligned}
\zeta(t) &= Pr\left[i_2(t) \leq n(t)\right] = \\
&= Pr\left[a^{\frac{[r+(1-n^c)/\varpi]P(t)}{\iota(t)}} \leq n(t)\right] \\
\nu(t) &= Pr\left[\tau w b x_1(t) \geq D(t)(1+\iota(t))\right] \\
&= Pr\left[\tau w b g(k_1(t)) \geq D(t)(1+\iota(t))\right] \\
&= Pr\left\{\tau w b g\left[k_1(t) + a^{\frac{(r+(1-n^c)/\varpi)P(t)}{\iota(t)}}\right] \geq D(t)(1+\iota(t))\right\}
\end{aligned}$$

with $\varpi = \mathbb{E}[\tilde{\varpi}]$.

- Denoting by η the *a-priori* probability for a firm to be in state 1, transition rates will be given by:

$$\lambda(t) = (1-\eta)\zeta(t) \tag{6}$$

$$\mu(t) = \eta\nu(t) \tag{7}$$

3.2 System dynamics

Chapman-Kolmogorov or **master equation**: quantifies the variation of probability flows in a small interval of time:

$$\frac{dP(N_z,t)}{dt} = \text{(inflows of probability fluxes into state } z\text{)-(outflows of probability fluxes out of state } z\text{)}$$

$$\frac{dp(N_z,t)}{dt} = \lambda p(N_z - 1)(t) + \mu p(N_z + 1)(t) - \{[(\lambda + \mu)p(N_z)(t)]\} \tag{8}$$

3.3 Analytical solutions

Evaluating the components of the dynamics

1. Split the state variable N_1 into two components:
   - the *drift* (m): trend value of the mean of $n_1 = N_1/N$; the *spread* (s): aggregate fluctuations around the drift;
   - hypothesis:
   $$N_1 := Nm + \sqrt{N}s \tag{9}$$
2. Use of lead and lag operators to homogenize in and out transition fluxes;
3. Taylor's expansion of the modified master equation;
4. Equating the terms with same order of power for N.

Asymptotic solution: dynamics

- An *o.d.e.* for the drift of the stochastic process:

$$\frac{dm}{dt} = \lambda m - (\lambda + \mu)m^2 \tag{10}$$

- *Fokker-Planck equation* for the transition density $Q(s, \tau)$ of the spread:

$$\begin{aligned} \frac{\partial Q}{\partial \tau} = \quad & [2(\lambda + \mu)m - \lambda] \frac{\partial}{\partial s}(sQ(s, \tau)) + \\ & + \frac{\left[\lambda m(1 - m) + \mu m^2\right]}{2} \left(\frac{\partial}{\partial s}\right)^2 Q(s, \tau) \end{aligned} \tag{11}$$

Asymptotic solutions: stationary equilibrium

- Trend dynamics and stationary state:

$$\boxed{m(t) = \frac{\lambda}{(\lambda+\mu)-\kappa e^{-\psi(t)}} \;\Rightarrow\; m^* = \frac{\lambda}{\lambda+\mu}} \tag{12}$$

where: $\kappa = 1 - \frac{m^*}{m(0)}$, $\psi = \frac{(\lambda+\mu)^2}{\lambda}$.

- Long run probability density of fluctuations ($\lim_{\tau\to\infty} Q(s,\tau) = \bar{Q}$):

$$\boxed{\bar{Q}(s) = C\exp\left(-\frac{s^2}{2\sigma^2}\right) \;:\; \sigma^2 = m^*\frac{\mu}{\lambda+\mu}} \tag{13}$$

4 Solution

The dynamics of the economy is then governed by the following system:

$$\begin{cases} dn_1(t) = (\lambda n_1(t) - (\lambda + \mu)[n_1(t)]^2)dt + \sigma\ dW \\ dK(t) = dI(t)dt = N\left\{[aP_{k1}(t)]n_1(t) + [aP_{k2}(t)][1 - n_1(t)]\right\} dt \end{cases} \tag{14}$$

where $\sigma\ \ dW$ are the stochastic fluctuations in the number of speculative firms, coming from the distribution (13).

5 Simulations

We use the following set of parameters:

- $n^c \in [0, 1]$;
- The values of ρ_z are the median of the ρ^js within each cluster of firms;
- $\tilde{\varpi} \in [0.1, 2]$;
- $a = 0.8$;
- $\phi = 0.4$;
- $\iota(0) = 0.1$; $c = 20$;
- the functions for investors' allocations are specified as

$$\begin{cases} \epsilon_1(t) = \frac{1}{1+exp(\iota(t)+\rho_2(t)-\rho_1(t))} \\ \epsilon_2(t) = \frac{1}{1+exp(\iota(t)+\rho_1(t)-\rho_2(t))} \\ \beta(t) = 1 - \epsilon_1(t) - \epsilon_2(t) \end{cases} \tag{15}$$

Figure 1: Different dynamics of capital.

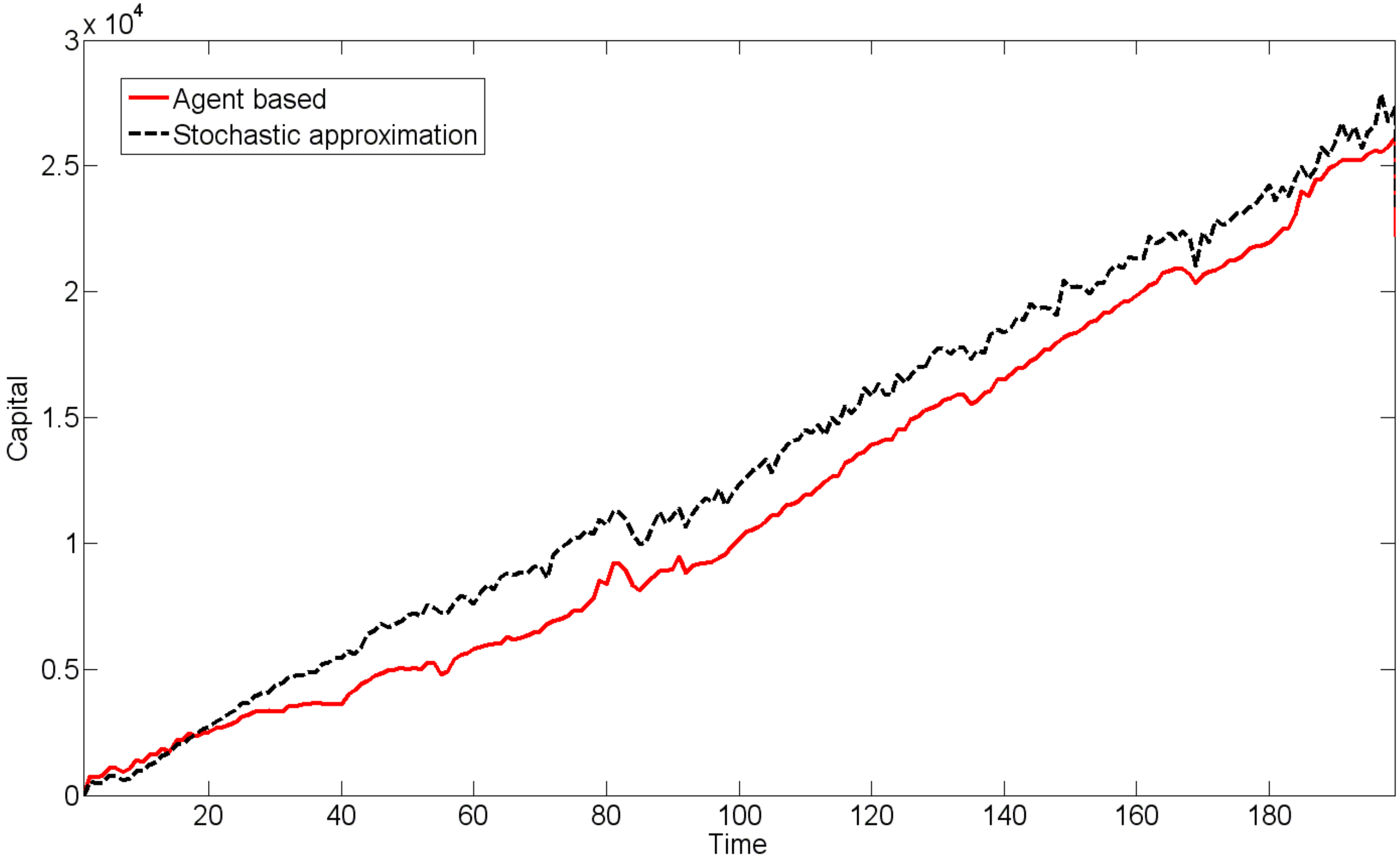


Figure 2: Different dynamics of the proportion of speculative firms.

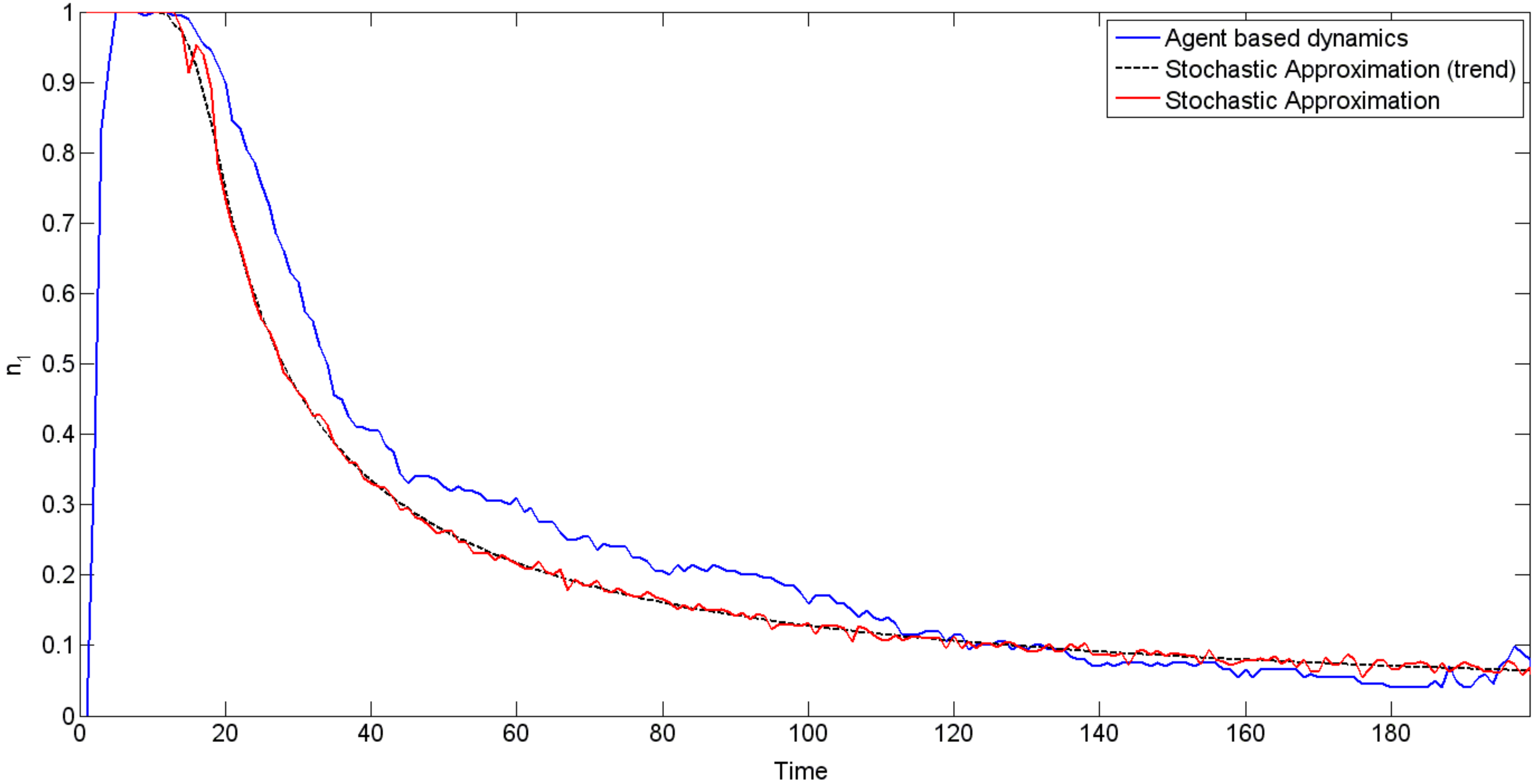


Figure 3: Evolution of firms' capital distribution.

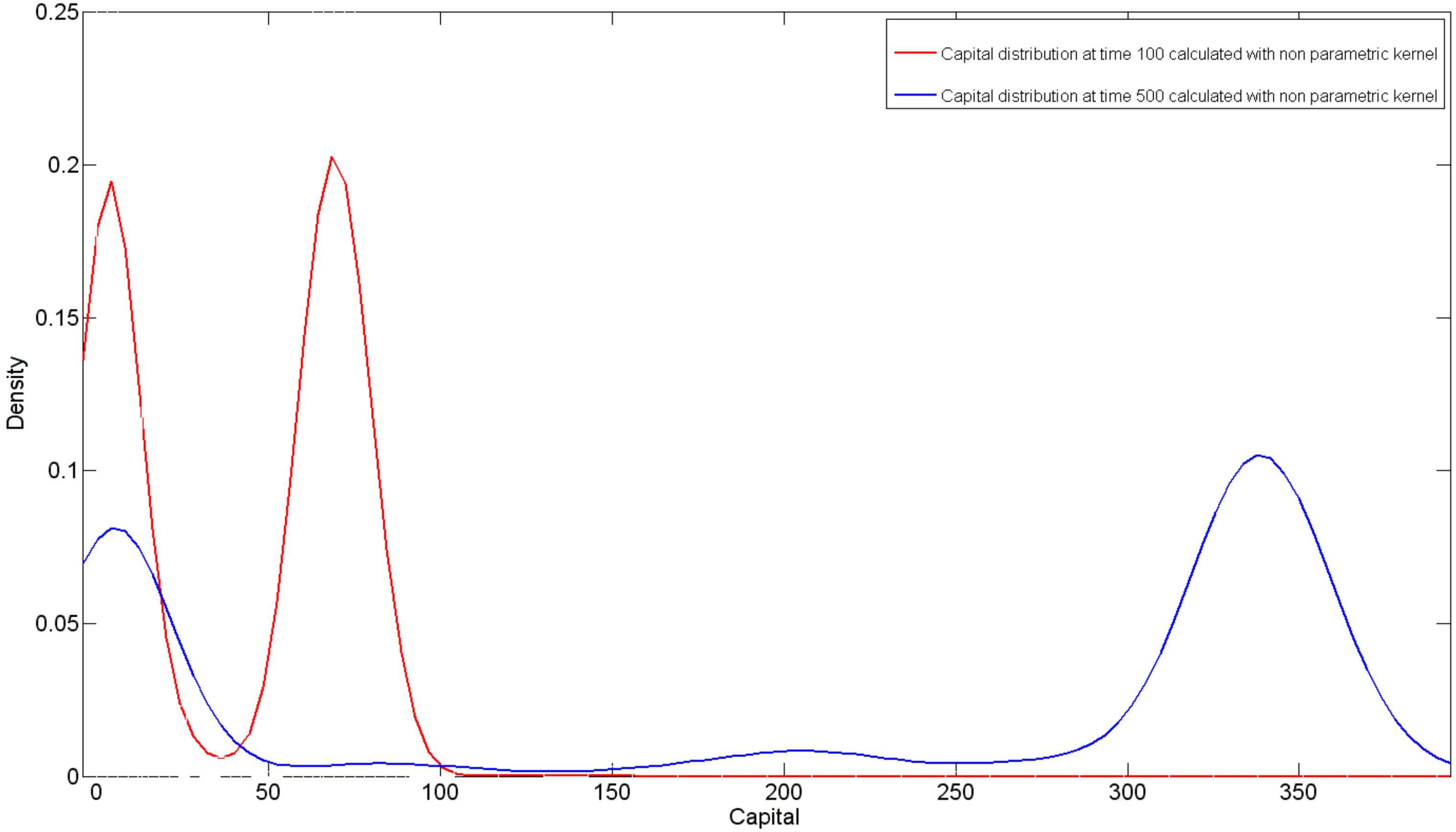


Figure 4: Debt distribution during a debt cycle.

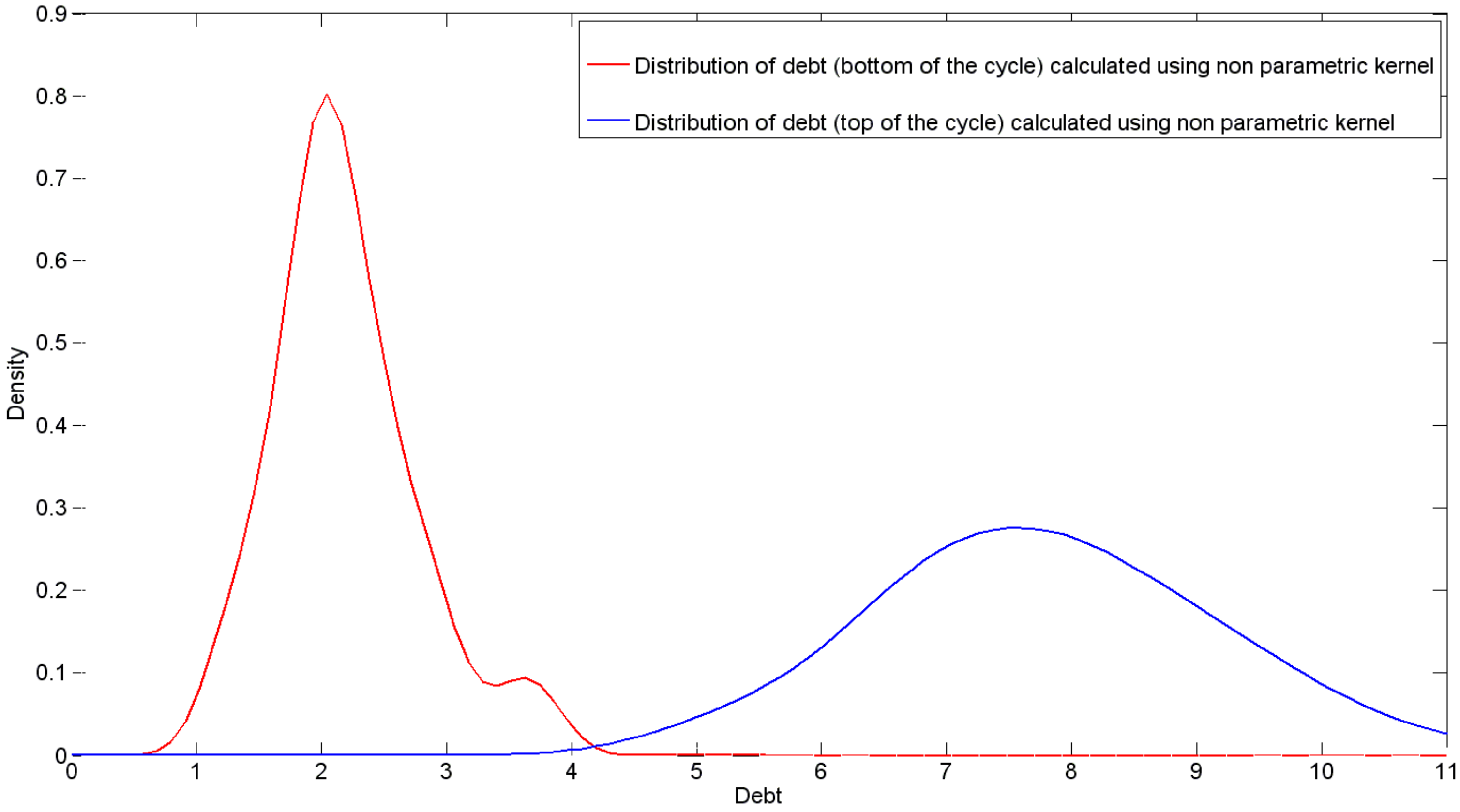


Figure 5: Capital distribution during a debt cycle.

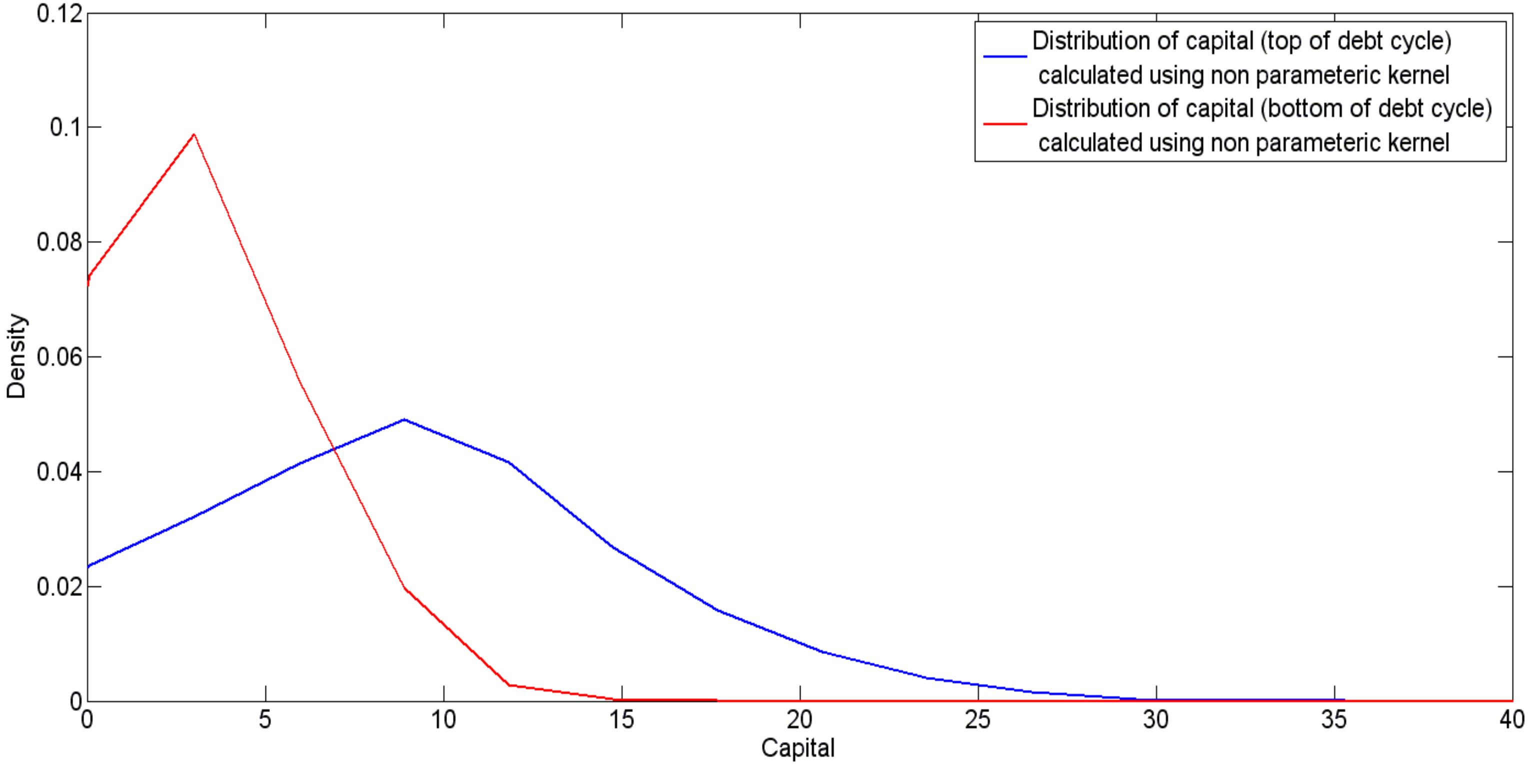
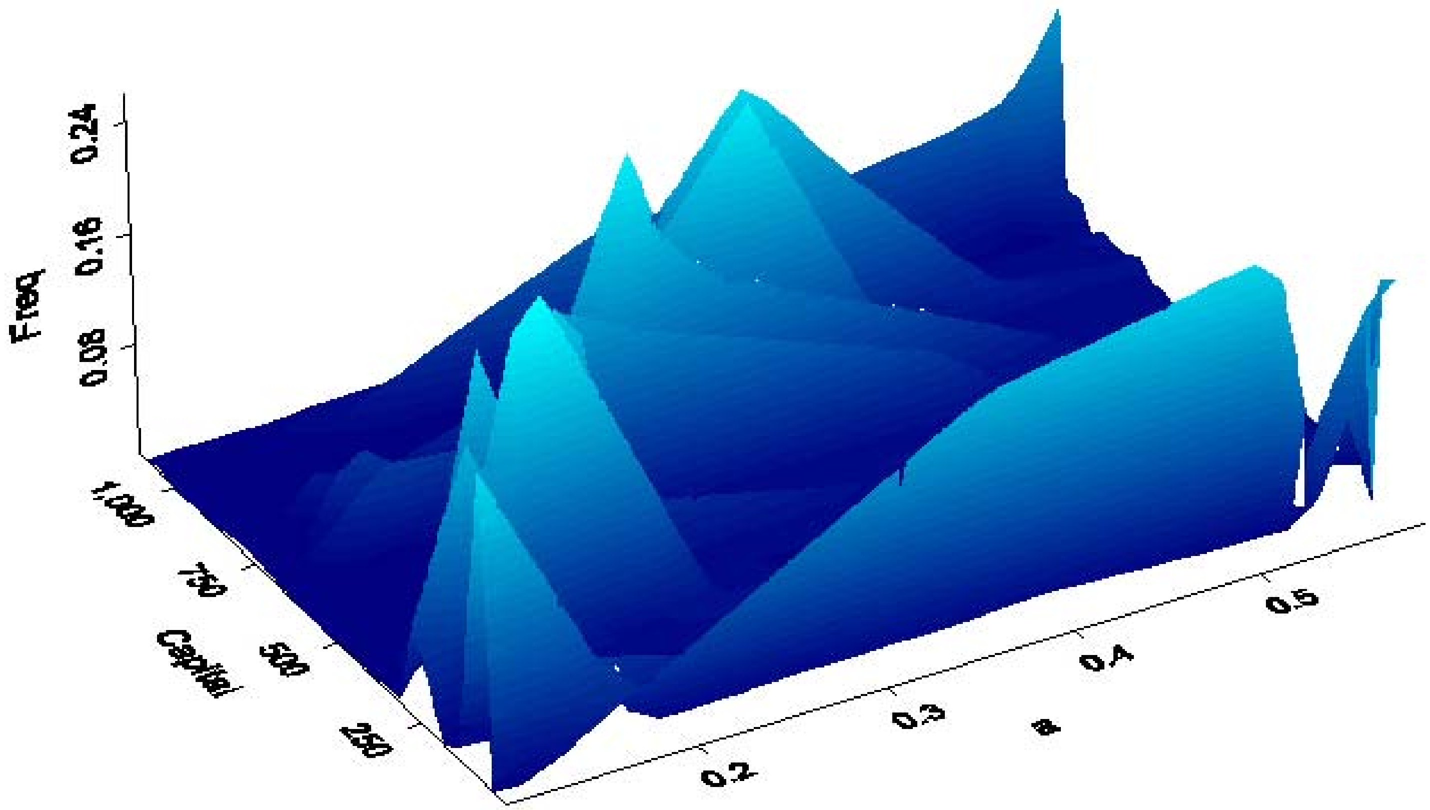


Figure 6: Frequency distributions of capital for different a.



6 Concluding remarks

- **Results**:
  - a consistent microfoudation of Minsky's investment framework;
  - a model with heterogeneous agents for which an approximated analytical solution can be obtained:
    * results satisfactorily mimic the outcomes of an agent based model with a much higher degree of heterogeneity;
  - a tool to analyse the effect of instability in financial markets on the real sector of the economy.

- **Future research**:
  - a deeper statistical analysis of the whole range of possible outcomes;
  - the identification of the conditions under which the system generates speculative bubbles and how they burst;
  - a more refined modelling of financial markets;
  - the study of the effects of:
    - additional hypotheses on firms (*e. g.* the possibility of buying their own shares);
    - high leverage;
    - various forms of speculative behaviour;
    - the introduction of a banking sector;
    - government policies:
      - fiscal, monetary;
      - regulatory framework.

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