Financial Fragility and Fluctuations in a World With Multi-Heterogeneous Agents

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The complexity of financial crisis in a long-period perspective: facts, theory and models
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Outline

Motivations

- The paper extends an approach to financial fragility analysis initiated by **Vercelli (2000)** and further developed by **Sordi & Vercelli (2006)** and by **Dieci, Sordi & Vercelli (2006)**, where financial fluctuations are the result of the dynamic interaction between current and inter-temporal financial ratios.
- The model considered in the previous contributions is a simple prototype model that is proposed in order to describe the complex dynamics of a sophisticated monetary economy, i.e., an economy that has fully developed financial infrastructures and interrelations.

Motivations

- It is a widely recognized fact that the financial constraints and objectives of economic agents have assumed a crucial role in shaping their behaviour.
- The analysis of the financial determinants of economic behaviour is therefore becoming a general issue that affects the entire economy. In this case it is reasonable to model all decision makers as financial units, focusing on the interaction between their **current** and **intertemporal financial constraints**.

The Model

- The main novelty of this work is the attempt to provide micro- economic foundations to the model, in order to better understand the complex dynamic behaviour generated in the above mentioned works and its policy implications
- The framework is modelled as a multi-heterogeneous agent model which proceeds through discrete time steps within a finite time horizon.
- The agents take decisions on the basis of limited rationality constraints
  - they are liable to systematic mistakes in forming their expectations

Agent-based model

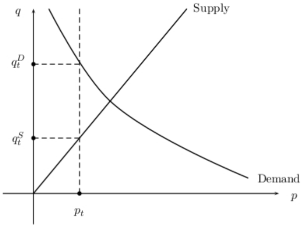
- Our model proceeds through discrete timesteps. A set of N financial units (each labelled by a roman index $i = 1, \cdots, N$) interacts at each timestep t.
- Financial units are heterogeneous in terms of their exchange strategies for the only risky asset in the economy.

Demand and Supply curves

We assume linear supply curve and hyperbolic demand curve for each financial unit

$$p = f(q^S) \rightarrow p_t = \alpha_{i,t} q^S_{i,t} \Rightarrow q^S_{i,t} = \frac{p_t}{\alpha_{i,t}}$$

$$p = f(q^D) \rightarrow p_t = \frac{\beta_{i,t}}{q^D_{i,t}} \Rightarrow q^D_{i,t} = \frac{\beta_{i,t}}{p_t}$$



Individual current inflow and outflow

Thus, the individual current inflow (y_i) and outflow (e_i) at time t are given by

$$y_{i,t} = p_t q^S_{i,t} = \frac{p_t^2}{\alpha_{i,t}},$$

$$e_{i,t} = p_t q^D_{i,t} = \beta_{i,t},$$

Therefore, the current realized financial ratio of the financial unit i $(c_{i,t})$ at time t is given by

$$c_{i,t} = \frac{e_{i,t}}{y_{i,t}} = \frac{\alpha_{i,t}\beta_{i.t}}{p_t^2}.$$

Market Maker

The individual demand and supply quantities of the N units are aggregated according to

$$D_t = \sum_{i=1}^{N} q_{i,t}^{D}, \quad S_t = \sum_{i=1}^{N} q_{i,t}^{S}.$$

Once D_t and S_t have been computed, the market maker clears the market by taking an offsetting position and computes the price for the next period by using the rule

$$p_{t+1} = p_t + \lambda_M \left(\frac{D_t - S_t}{D_t} \right)$$

where λ_M is the market maker's speed of adjument of the price to excess demand.

Desired and Intertemporal ratios

Each financial unit i, at each time t, computes its desired current financial ratio $\left(k_{i,t}\right)$, on the basis of the recursive relationship (see Vercelli (2000), Sordi & Vercelli (2006), Dieci et al. (2006))

$$k_{i,t+1} = \max\left\{k_{i,t} - \lambda_{k_i}\left[k^*_{i,t} - \left(1 - \mu^i\right)\right], 0\right\},$$

where the intertemporal financial ratio of unit i $\left(k^*_{i,t}\right)$ at time t is given by

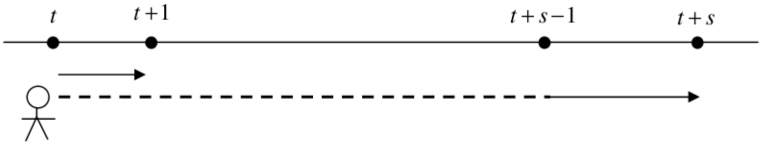
$$k^*_{i,t} = \frac{\sum_{s=0}^{T_i} \mathbb{E}_t\left[e_{i,t+s}\right] / \left(1+\rho_{i,t}\right)^s}{\sum_{s=0}^{T_i} \mathbb{E}_t\left[y_{i,t+s}\right] / \left(1+\rho_{i,t}\right)^s},$$

with $\rho_{i,t}$ being the discount factor for the financial unit i at time t and T_i is its time horizon.

Expected inflation

When agents come to form expectations about inflows and outflows in future periods we assume that they expect these to grow by the accumulation of their expectation of inflation over the next period.

$$\pi^{i}_{t|t+s-1,t+s} = \mathbb{E}^{(i)}_{t}\left[\frac{p_{t+s}-p_{t+s-1}}{p_{t+s-1}}\right] = \begin{cases}\text{the expectation at time } t \\ \text{of inflation over the period} \\ \quad (t+s-1, t+s)\end{cases}$$



We assume that the relation between individual discount factors, nominal interest rate r_t and the individual expected inflation rate from t to $t+1$ has the form

$$1+\rho_{i,t} = (1+r_t)\left(1+\pi^{i}_{t,t+1}\right)$$

Agent i assumes $\pi_{t|t+s-1,t+s} = \pi^{i}_{t|t,t+1} \equiv \pi^{i}_{t,t+1}$.
Then, we can rewrite k^* as follow

$$k^*_{i,t} = \frac{\sum_{s=0}^{T_i} \mathbb{E}_t\left[e_{i,t+s}\right] / \left(1+\rho_{i,t}\right)^s}{\sum_{s=0}^{T_i} \mathbb{E}_t\left[y_{i,t+s}\right] / \left(1+\rho_{i,t}\right)^s}$$

$$= c_{i,t}\left[\frac{(1+r_t)^{T_i+1}-1}{r_t}\right]\left[\frac{r_t-\pi^{i}_{t,t+1}}{(1+r_t)^{T_i+1}-(1+\pi^{i}_{t,t+1})^{T_i+1}}\right]$$

Expected Inflation

After the market maker announces p_{t+1}, each financial unit updates its expected inflation for the next period according to the simple adaptive rule

$$\begin{aligned} \pi^i_{t,t+1} &= \pi^i_{t-1,t} + \lambda^i_\pi \left(\frac{p_t - p_{t-1}}{p_{t-1}} - \pi^i_{t-1,t} \right) \\ &= \lambda^i_\pi \left(\frac{p_t - p_{t-1}}{p_{t-1}} \right) + \left(1 - \lambda^i_\pi \right) \pi^i_{t-1,t}. \end{aligned}$$

Demand and Supply curves

The desired financial ratio $k_{i,t+1}$ affects the demand and supply curves in the following way

$$\beta_{i,t+1} = k_{i,t+1}\beta_{i,t},$$
$$\alpha_{i,t+1} = k_{i,t+1}\alpha_{i,t}.$$

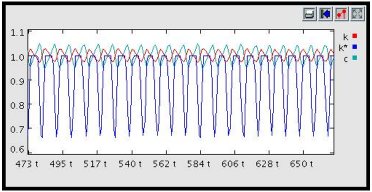
Parameters

The agent-based framework presented in this paper has been implemented in Java using JAS library.
We performed several simulation under the following parameters configuration:

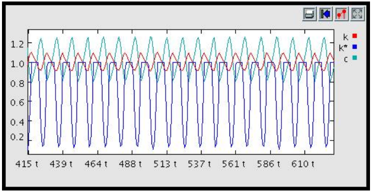
- $p_0 = 1.2$;
- $r = 0.05$;
- $\pi_0^i = 0.04$;
- $\lambda_k^i = 0.04$;
- $\lambda_\pi^i = 0.5$;

Test 1: Financial Ratios - $\lambda_M = 0.3$

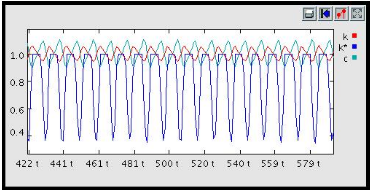
$1 - \mu = 0.9$



$1 - \mu = 0.7$

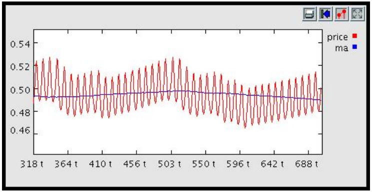


$1 - \mu = 0.8$

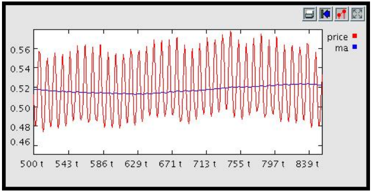


Test 1: Price impact - $\lambda_M = 0.3$

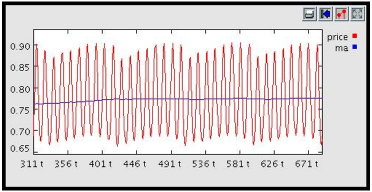
$1 - \mu = 0.9$



$1 - \mu = 0.8$

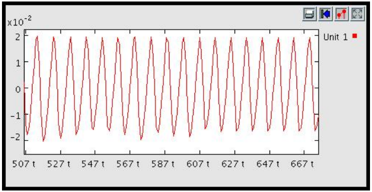


$1 - \mu = 0.7$

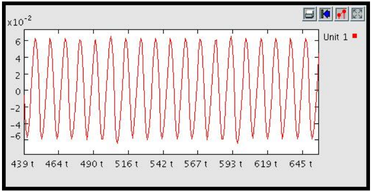


Test 1: Expected inflation rate - $\lambda_M = 0.3$

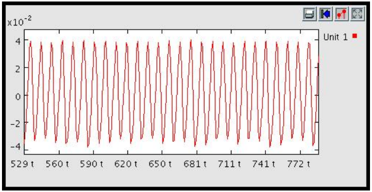
$1 - \mu = 0.9$



$1 - \mu = 0.7$

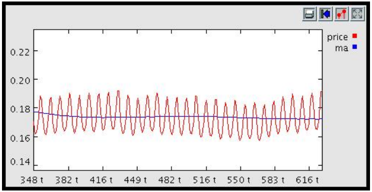


$1 - \mu = 0.8$

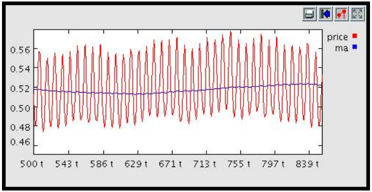


Test 2: Price - $1 - \mu = 0.8$

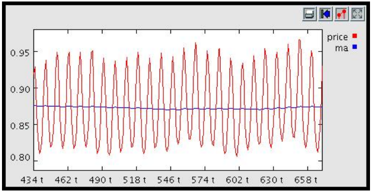
$\lambda_{MM} = 0.1$



$\lambda_{MM} = 0.3$

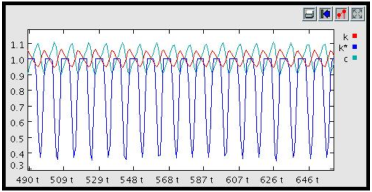


$\lambda_{MM} = 0.5$

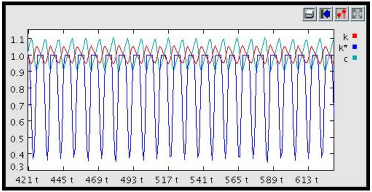


Test 2: Financial ratios - $1 - \mu = 0.8$

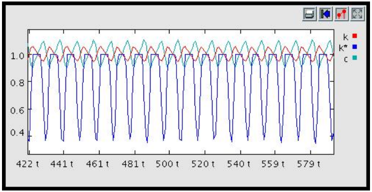
$\lambda_{MM} = 0.1$



$\lambda_{MM} = 0.5$

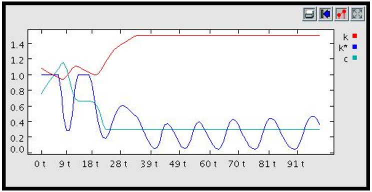


$\lambda_{MM} = 0.3$

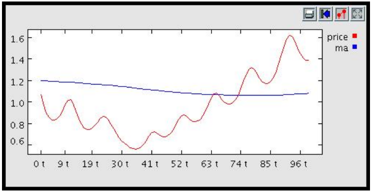


Test 3: two units - $1 - \mu^1 = 0.8$, $1 - \mu^2 = 0.75$ and $\lambda_{MM} = 0.3$

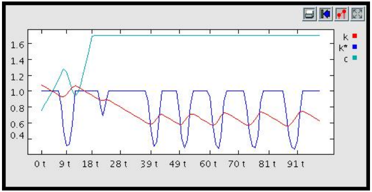
Unit 1 - Financial ratios



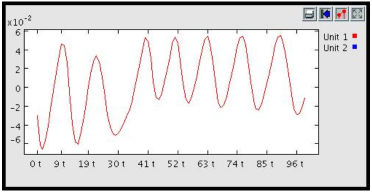
Market price



Unit 2 - Financial ratios



Individual expected inflation



Conclusions

- We exdends the macro-financial frigility model presented in **Dieci, Sordi & Vercelli (2006)**, providing the micro-foundation of the model
- The framework has been modelled as an heterogeneous agent model
- The financial unit decision is not fully rational

Further developments

- Introduce budget constraint and borrowing
- Endogenize the evolution of r based on different monetary policies
- Endogenize the individual financial fragility

Thank you for your attention!!!