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# ENDOGENOUS GROWTH AND NEGATIVE EXTERNALITIES

ABSTRACT: We augment an AK model by treating the units of time devoted to work as a choice variable and by introducing an environmental resource entering the households' utility function. In general, the resulting model does not generate endogenous growth in the absence of negative externalities: perpetual growth can be generated only when the resource deteriorates because of the consumer activities. In this case, indeed, the households keep their labor supply and saving rates relatively high in spite of their increasing private wealth in order to consume more private goods as substitutes for the declining quality of the environment.

KEY WORDS: Common property, Defensive expenditure, Environmental assets.

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## 1. Introduction*

The aim of this paper is to present a view on growth which differs from the dominant paradigm, with its insistence that unbounded growth is fuelled by positive externalities. We instead emphasize the role played in the growth process by negative externalities: the expansion of consumption erodes the quality and reduces the endowment of resources to which all individuals have free access, thereby forcing them to increase their dependence on private goods in order to satisfy their needs. This boosts production and feeds the growth process.

To present this view on growth, we augment a Ramsey-Rebelo AK model by treating the units of time devoted to work as a choice variable. In general, the resulting model does not generate endogenous growth in the absence of negative externalities, i.e. when consumption has no effect on a renewable resource.[1] In this case, indeed, the return on capital investment is reduced by the less time devoted by individuals to work as the capital stock grows larger and the households become richer. In contrast, as the renewable resource deteriorates because of the pollution caused by the consumers' activities, households seek to defend their welfare against this deterioration by consuming more private goods. Moreover, they anticipate that the process of environmental degradation will go on in the future, thus forcing them to rely increasingly on man-made goods. Hence, the households are induced to keep both their labor effort and saving rate relatively high, despite their rising private wealth. Unbounded growth is the result of this process.

Although the aggregate impact of the households' consumer activities on environmental quality is remarkable, the detrimental impact of each single household's activity is negligible. Thus, in the lack of well-defined property rights on the resource or of a regulatory authority imposing taxes and subsidies, there is no incentive for the households to internalize the negative externalities that they generate: the equilibrium

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[1] In Rebelo's (1991) AK model , the absence of diminishing returns to capital can be made plausible by interpreting K in a broad sense to include human capital (see also Barro and Sala-i-Martin, 1995). It is even more plausible to let labor enter the production function both as a reproducible factor whose quality depends on previous investment and as an input whose quantity depends on current choices. In other words, physical capital tends to increase together with the quality of the working population, but not necessarily with the time that this population devotes to work.

path of the economy is not Pareto-optimal.

This perspective on the growth process is based on ideas with a long and interdisciplinary history behind them.[2] Legitimate interpretations of the mechanism described in this paper can be formulated both in terms of the damage wrought to environmental assets by consumer activities, and in terms of the undermining of the institutional and non-material bases of communal sources of welfare by the increasing dominance of individualistic modes of consumption. In both cases, individuals must increasingly rely on private goods in order to avert a drastic decline in their well-being. We share with the literature on sustainable growth its concern for the potential impact that current economic activities may have – by depleting social and environmental assets – on long-term growth performances and future well-being. The emphasis in the literature, however, is on whether unbounded growth is possible in the presence of natural resources negatively affected by the growth process,[3] and not on the role as the 'engine' of growth played by the progressive degradation of these assets. This literature does not seem entirely aware of the extent to which the declining endowment of free resources is able to boost economic growth. By contrast, we focus precisely on the manner in which work attitudes, saving propensity and consumption habits become more favorable to growth as access to free resources diminishes.

In recent years, some authors have investigated the implications of including the determination of leisure and effort in an endogenous growth model. In particular, Duranton (2001) is of special interest for our framework.[4] Indeed, he demonstrates that, as long as the demand for leisure increases in the income,

[2] For a discussion of some of these ideas see Bartolini and Bonatti (1999).

[3] In Musu (1994), sustainable growth is consistent with a modified AK model, assuming that there is no increase in pollution as production increases because of higher capital stock. In the absence of a lower limit, below which environmental quality cannot fall without entailing irreversible catastrophe, Martin and Rotillon (1996) analyse under what conditions on the utility function the AK model is able to generate sustainable growth. In the presence of such a limit, Aghion and Howitt (1998) show that growth is not sustainable with a AK production function. In a AK model with a utility cost of pollution (which increases linearly with the output) and a binding emission standard, Stokey (1998) shows that perpetual growth is possible even if the emission standard gets stricter and total pollution falls over time.

[4] Endogenous growth models which treat leisure as a choice variable but are not directly relevant for our framework include Ladrón-de-Guevara et al. (1999) and Ortigueira (2000). In these models, time can be devoted to production of goods, to education and to leisure. Ladrón-de-Guevara et al. (1999) show that multiple steady-state equilibria exist in a

sustained growth is driven to an end, since households tend to work less when wealth accumulates. Hence, he points out the contradiction arising from the fact that growth still occurs in the real world although for realistic assumptions on agents' preferences his model predicts that it would stop. Consequently, Duranton (2001) concludes by indicating the need to explore some mechanisms that may explain the persistence of high labor supply when the economy grows richer, thus restoring the possibility of unbounded growth. One of these mechanisms has been emphasized in the papers by Cole et al. (1992), Konrad (1992), Robson (1992), Corneo and Jeanne (1995), which study the implications for saving and capital accumulation of the Veblen's hypothesis that individuals care about their social status, as determined by their relative wealth. The present paper aims at offering an alternative explanation stressing how the degradation of environmental and social assets brought about by the growth process can induce the individuals to work and save more in order to buy more private goods as substitutes for the declining endowment of free resources.

This paper is organized as it follows. Section 2 presents the model and derives the optimizing behavior of the agents. Section 3 characterizes the equilibrium paths of the economy, showing that it is only in the presence of negative externalities that perpetual growth is possible. Section 4 concludes.

## 2. The model and the optimizing behavior of agents

We consider an economy in discrete time with an infinite time horizon. For simplicity and without loss of generality, it is assumed that population is constant and that each household contains one adult, working member of the current generation. Thus, there is a fixed (and large) number I of identical adults who take account of the welfare and resources of their actual and perspective descendants. Indeed, following Barro and Sala-i-Martin (1995) we model this intergenerational interaction by imaging that the current generation maximizes utility and incorporates a budget constraint over an infinite future. That is, although individuals

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context where the quality of leisure does not change with the level of human capital. In a variant of this model where a certain amount of time devoted to leisure activities gives higher utility when agents have a greater stock of human capital, Ortigueira (2000) shows that there is a unique globally stable steady-state equilibrium.

have finite lives, we consider immortal extended families ("dynasties").[5] The current adults expect the size of their extended family to remain constant, since expectations are rational (in the sense that they are consistent with the true processes followed by the relevant variables). In this framework in which there is no source of random disturbances, this implies perfect foresight.

*Households' utility*

The period utility function of the representative household, $U_t$, increases in consumption and leisure:

$$U_t = u(x_t, h_t),\ u_{x_t} > 0,\ u_{x_t x_t} < 0,\ u_{h_t} < 0,\ u_{h_t h_t} < 0,\ u_{h_t x_t} \le 0,\ 0 \le h_t \le 1, \tag{1}$$

where $x_t$ is the amount of services generated by a consumer activity in period t, and $h_t$ are the units of time spent working in t by the household (the total amount of time available to each household in period t is normalized to be one). Households generate $x_t$ by adopting a consumer technology that combines a resource to which all individuals have free access in every period and a consumer good that can be privately appropriated:[6]

$$x_t = x(R_t, C_t) = (R_t C_t)^{\delta},\ 0 < \delta \le 1, R_t \ge 0, C_t \ge 0, \tag{2}$$

where $R_t$ is the endowment (or an index of the quality) in t of a free resource that cannot be produced, and $C_t$ is the amount of the unique good produced in this economy that is devoted to consumption in t. Note that there is non-rivalry in the consumers' use of the resource $R_t$, from which no consumer can be excluded: it has the nonexclusive nature typical of a public good. Moreover, it is worth to emphasize that $R_t$ and $C_t$ are complements in the production of $x_t$, in the sense that the marginal (consumer) production function $\frac{\partial}{\partial C_t} x(R_t, C_t)$ is increasing in $R_t$ holding $C_t$ fixed. Given $u_{x_t x_t} < 0$, this implies that the degradation of the

[5] As Barro and Sala-i-Martin (1995, p. 60) point out, "this setting is appropriate if altruistic parents provide transfers to their children, who give in turn to their children, and so on. The immortal family corresponds to finite-lived indiiduals who are connected via a pattern of operative intergenerational transfers that are based on altruism".

[6] In the household production function approach, the quality of a household's personal environment is treated as a function of the quality of the collective environment and of goods that can be privately appropriated. For applications of this approach to measuring the demand for environmental attributes, see Kerry Smith (1991).

environment can prevent marginal utility of $x_t$ from falling as $C_t$ rises.

*Production*

There is only one good $Y_t$ produced in this economy. Each household produces this single good according to the technology

$$Y_t = AK_t h_t^{\alpha}, \ A > 0, 0 < \alpha \leq 1, \ K_t \geq 0, \tag{3}$$

where A is a parameter measuring the state of technology, $K_t$ is the stock of capital existing in t (capital can be interpreted in a broad sense, inclusive of all reproducible assets).

*Capital*

The stock of capital evolves according to

$$K_{t+1}=Y_t+(1-\sigma)K_t-C_t, \ 0<\sigma<1, \ K_0 \text{ given}, \tag{4}$$

where $\sigma$ is a capital depreciation parameter.

*Free resource*

We take into consideration two possible cases: the first case deals with the situation in which the evolution in time of the free resource is not affected by the households' activities, while in the second case the ability of the free resource to regenerate declines with the level of consumers' activities.

In the first case, we assume that the resource evolves according to the logistic model, which is one of the simplest and best known functional specification for the law of motion of a renewable resource (see Conrad, 1987):

$$R_{t+1} - R_t = rR_t\left(1 - \frac{R_t}{E}\right), \ 0 < r \leq 2, E > 0, \ R_0 \text{ given}, \tag{5a}$$

where the parameters r and E can be interpreted as, respectively, the intrinsic growth rate[7] and the environmental carrying capacity.

In the second case, we modify the logistic specification by assuming that environmental quality

[7] The restriction $0<r\leq 2$ ensures that $R_t$ will approach asymptotically its steady-state value E.

declines whenever the pollution generated by the consumers' activities surpasses the environmental carrying capacity:

$$R_{t+1} - R_t = rR_t\left(1 - \frac{Ip_t}{E}\right),\ 0 < r \le 2,\ E > 0,\ R_0 \text{ given,} \tag{5b}$$

where $p_t$ is the level of pollution generated in t by each household. Total pollution increases with the number of households and with the quantity of consumer services produced in t by each household:

$$p_t = \rho x_t^{\gamma},\ \gamma > 0,\ \rho > 0.^{8} \tag{6}$$

*Households' objective*

In each period, the representative household must decide on $h_t$ and $C_t$ in order to maximize its discounted sequence of utilities:

$$\sum_{i=0}^{\infty} \theta^i U_{t+i},\ 0<\theta<1, \tag{7}$$

where $\theta$ is a time preference parameter.

*Optimizing behavior*

In the case in which the motion of $R_t$ is governed by (5b), each single household can ignore the negative impact of her consumer activity on the future environmental quality, since its own contribution to the generation of total pollution is negligible. In this case, indeed, the impact of the consumer activities on the future endowment of natural resource is significant because of the large number of households populating the economy. Therefore, no matters whether the motion of $R_t$ is governed by (5a) or by (5b), the problem of each household amounts to maximize the Hamiltonian

$H_t = \sum_{i=0}^{\infty} \theta^i \left\{U_{t+i} - \lambda_{t+i}\left[K_{t+i+1} - AK_{t+i}h_{t+i}^{\alpha} - (1-\sigma)K_{t+i} + C_{t+i}\right]\right\}$ with respect to $C_t$, $h_t$ and $K_{t+1}$, where

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[8] Considering (2) and (6), the consumer production function can be rewritten as $x_t = \min\left((R_t C_t)^{\delta}, \left(p_t/\rho\right)^{1/\gamma}\right)$ (see Smulders, 2000).

$\lambda_{t+i}$ is the multiplier. Hence, one obtains the conditions that each household must satisfy:

$$\delta R_t^{\delta} C_t^{\delta-1} u_{x_t} = \lambda_t , \tag{8a}$$

$$-u_{h_t} = A\alpha K_t h_t^{\alpha-1} \lambda_t , \tag{8b}$$

$$\lambda_t = \lambda_{t+1}\theta\left(Ah_{t+1}^{\alpha} + 1 - \sigma\right) \tag{8c}$$

A path maximizing (7) must also satisfy the laws of motion (4) and (5a) or (5b), and the transversality condition

$$\lim_{t\to\infty} \theta^{t} \lambda_t K_t = 0. \tag{9}$$

It is straightforward that a path satisfying (8) and (9) is not Pareto-optimal when the motion of $R_t$ is governed by (5b) (see the Appendix for the conditions to be satisfied by a Pareto-optimal path in the presence of negative externalities).

## 3. Equilibrium paths

In this section we give an example where in general the economy can achieve a strictly positive long-run growth rate only if there are negative externalities, i.e. only if the motion of $R_t$ is governed by (5b). In this example, the households' utility function is additively separable between consumption and leisure ( $u_{h_t x_t} = 0$). However, in the Appendix we give another example where $x_t$ and $1-h_t$ are complements (in the sense that $u_{h_t x_t} < 0$): also in this case, a balanced growth path characterized by a strictly positive rate of growth can exist only if there are negative externalities.

Let us assume that

$$u(x_t, h_t) = \beta \frac{(x_t)^{1-\xi}}{(1-\xi)} + (1-\beta)\frac{(1-h_t)^{1-\zeta}}{(1-\zeta)}, \quad \xi > 0,\ \zeta > 0,\ \ 0 < \beta < 1. \tag{10}$$

Given (10), one can use (4) and (8) to obtain the system of equations that – together with (5a) or (5b) -- governs this economy:

$$\frac{K_{t+1} h_{t+1}^{\alpha-1}(1-h_{t+1})^{\zeta}}{\theta\left(Ah_{t+1}^{\alpha} + 1 - \sigma\right)} = K_t h_t^{\alpha-1}(1-h_t)^{\zeta} , \tag{11a}$$

$$K_{t+1} = AK_t h_t^{\alpha} + K_t(1-\sigma) - C_t, \tag{11b}$$

where

$$C_t = \left[ \frac{A\alpha\beta\delta(1-h_t)^{\zeta} K_t R_t^{\delta(1-\xi)}}{(1-\beta)h_t^{1-\alpha}} \right]^{1/[1-\delta(1-\xi)]}. \tag{11c}$$

Note that equations (11a) and (11b) can be rewritten as

$$\frac{(1+\mu_t)h_{t+1}^{\alpha-1}(1-h_{t+1})^{\zeta}}{\theta\left(Ah_{t+1}^{\alpha}+1-\sigma\right)} = h_t^{\alpha-1}(1-h_t)^{\zeta}, \;\; \mu_t \equiv \frac{K_{t+1}-K_t}{K_t}, \tag{12a}$$

$$1+\mu_t = Ah_t^{\alpha} + (1-\sigma) - \left[ \frac{A\alpha\beta\delta h_t^{\alpha-1}(1-h_t)^{\zeta} Z_t^{\delta(1-\xi)}}{(1-\beta)} \right]^{1/[1-\delta(1-\xi)]}, \;\; Z_t \equiv R_t K_t. \tag{12b}$$

*The balanced growth path in the absence of negative externalities*

Equation (5a) can be rewritten as

$$\pi_t = r\left(1-\frac{R_t}{E}\right), \;\; \pi_t \equiv \frac{R_{t+1}-R_t}{R_t}. \tag{13a}$$

Equations (12) and (13a) governs the equilibrium path of the economy in the absence of negative externalities. An equilibrium path is a balanced growth path if $\mu_{t+1}=\mu_t=\mu$, $\pi_{t+1}=\pi_t=\pi$ and $h_{t+1}=h_t=h$ in equations (12) and (13a). Except for the special case in which households' preferences are such that $\xi=1$, a balanced growth path governed by (12) and (13a) must have $\mu_{t+1}=\mu_t=\mu=0$: in general, this economy cannot grow forever at a constant rate in the absence of negative externalities.

***Proposition 1****: In the general case in which $\xi\neq 1$, the economy whose motion is governed by (12) and (13a) cannot display perpetual growth.*

*Proof:* The proof amounts to show that a balanced growth path must be such that $\mu_{t+1}=\mu_t=\mu=0$ and that an equilibrium trajectory must converge to a balanced growth path. (i) By inspecting (13a), it is apparent that along a balanced growth path one must have $\pi_{t+1}=\pi_t=\pi=0$ and $R_{t+1}=R_t=R=E$. (ii) By inspecting (12b) in the general case in which $\xi\neq 1$, it is apparent that along a balanced growth path one must have $Z_{t+1}=Z_t=Z$. (iii) Given that $Z_t\equiv R_t K_t$, it is apparent that $\mu_{t+1}=\mu_t=\mu\neq 0$ when $\xi\neq 1$ is inconsistent with the fact that both (i) and

(ii) must hold. Thus, in the general case in which $\xi \neq 1$, the balanced growth path of this economy without negative externalities is characterized by $\mu=0$ and $\pi=0$ (see the Appendix for the steady-state values of $R_t$, $K_t$ and $h_t$). Moreover, one can check that any trajectory which does not converge to the balanced growth path cannot be an equilibrium trajectory (see the Appendix).

The absence of unbounded growth is due to the fact that -- as the evolution of the public good is exogenously given -- it is not optimal in general for the households to allow capital to grow forever, even if the production function is such that for given levels of technology and labor effort the marginal productivity of capital does not decline as $K_t$ rises. This is because leisure can be substituted for consumption, and the return on capital investment is lowered by the shorter time that individuals will devote to work as the capital stock grows larger and the economy becomes more productive. In general, this economy can exhibit perpetual growth in the absence of negative externalities only if leisure does not enter the households' utility function ($\beta=1$): one can easily check that in this case (in which $h_t=1$ $\forall t$) along a balanced growth path one has $\mu = [\theta(\mathrm{A}+1-\sigma)]^{1/[1-\delta(1-\xi)]} - 1$.[9]

*The balanced growth path in the presence of negative externalities*

By using (2), (6) and (11c), equation (5b) can be rewritten as

$$\pi_t = r\left\{1 - \frac{I\rho}{E}\left[\frac{A\alpha\beta\delta h_t^{\alpha-1}(1-h_t)^{\zeta} Z_t}{(1-\beta)}\right]^{\gamma\delta/[1-\delta(1-\xi)]}\right\}. \tag{13b}$$

Equations (12) and (13b) governs the equilibrium path of the economy in the presence of negative externalities. An equilibrium path is a balanced growth path if $\mu_{t+1}=\mu_t=\mu$, $\pi_{t+1}=\pi_t=\pi$ and $h_{t+1}=h_t=h$ in equations (12) and (13b). The economy governed by by (12) and (13b) can grow forever at a constant rate.

***Proposition 2****: The economy whose motion is governed by (12) and (13b) can display perpetual growth.*

*Proof:* (i) By rewriting (13b) as

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[9] To satisfy the transversality condition when $\beta=1$, the parameters' values must be such that $\theta^{1/[1-\delta(1-\xi)]}(\mathrm{A}+1-\sigma)^{\delta(1-\xi)/[1-\delta(1-\xi)]} < 1$.

$$\frac{Z_{t+1}}{(1+\mu_t)} - Z_t = rZ_t\left\{1 - \frac{I\rho}{E}\left[\frac{A\alpha\beta\delta h_t^{\alpha-1}(1-h_t)^{\zeta} Z_t}{(1-\beta)}\right]^{\gamma\delta/[1-\delta(1-\xi)]}\right\}, \tag{13c}$$

the system governing the equilibrium path of the economy in the presence of negative externalities consists of three difference equations in $\mu_t$, $Z_t$ and $h_t$ ((12a), (12b) and (13c)). Hence, along a balanced growth path, one must have $\mu_{t+1}=\mu_t=\mu$, $Z_{t+1}=Z_t=Z$ $\left(\text{entaling} 1+\pi = 1/(1+\mu)\right)$ and $h_{t+1}=h_t=h$ in (12) and (13c). (ii) By solving the system consisting of (12) and (13c) for $\mu_{t+1}=\mu_t=\mu$, $Z_{t+1}=Z_t=Z$ and $h_{t+1}=h_t=h$, one can check that the solving triple $(\mu,Z,h)$ is such that in general $\mu\neq 0$ ($\mu=0$ only for particular combinations of parameter values).

Indeed, along a balanced growth path governed by (12) and (13c), one has:

$$\mu = \theta(Ah^{\alpha} + 1 - \sigma) - 1, \tag{14a}$$

$$Z = \frac{(1-\beta)h^{1-\alpha}}{A\alpha\beta\delta(1-h)^{\zeta}}\left\{\left[1 + r - \frac{1}{\theta(Ah^{\alpha}+1-\sigma)}\right]\left(\frac{E}{\rho Ir}\right)\right\}^{[1-\delta(1-\xi)]/\gamma\delta}, \tag{14b}$$

$$(1-\theta)(Ah^{\alpha}+1-\sigma) = \frac{A\alpha\beta\delta(1-h)^{\zeta}}{(1-\beta)h^{1-\alpha}}\left\{\left[1 + r - \frac{1}{\theta(Ah^{\alpha}+1-\sigma)}\right]\left(\frac{E}{\rho Ir}\right)\right\}^{(1-\xi)/\gamma}. \tag{14c}$$

It is apparent that $\mu=0$ if and only if the parameter values are such that $h = \left[\frac{1-\theta(1-\sigma)}{\theta A}\right]^{1/\alpha}$ satisfies (14c), which defines an implicit function $h=h(A,E,I,r,\alpha,\beta,\gamma,\delta,\theta,\rho,\sigma,\xi,\zeta)$. Along a balanced growth path with $\mu>0$, one has both $Y_t \to \infty$ and $R_t \to 0$ as $t \to \infty$: steady-state growth consists in the progressive substitution of a good that can be privately appropriated for a common property resource whose endowment is declining. Moreover, numerical examples show that the system which is obtained by linearizing (12) and (13c) around a triple $(\mu,Z,h)$ satisfying (14) can exhibit saddle-path stability (see the Appendix). Finally, it is worth to note that for having a unique balanced growth path is sufficient that $\xi\geq 1$. In other words, it is sufficient that $R_t$ and $C_t$ are not complements in consumption, namely that the marginal utility function $\frac{\partial}{\partial C_t}u(x(R_t,C_t),h_t)$ is not increasing in $R_t$ holding $C_t$ fixed. If $\xi\geq 1$, it is unambiguously the case that both a larger population size (larger I) and a greater impact of a given level of consumption on environmental quality (larger $\rho$/E) boost long-run per capita working time and output growth.[10] In fact, everything that exerts greater pressure on the environmental resource and accelerates its decline can induce individuals to react by working and saving more. Thus, according to the model, policies which reduce population growth and the environmental impact of consumer activities may restrain the long-term growth rate of per capita output.

## 4. Concluding remarks

An economy that increases its private wealth by accumulating capital keeps high the saving rate if households anticipate that the future endowment of the free resource will be negatively affected by the growth process, which induces them to increasingly substitute the private good for $R_t$ in their consumer activity. As the free resource deteriorates, the value of $C_t$ for households increases relatively to the value of time, and the return on capital investment is not depressed by the willingness of households to work less. The increasing labor productivity brought about by the rising capital stock is not used to reduce the time devoted to work, because the deterioration of $R_t$ makes it more urgent to increase private consumption. Acting entirely independently of each other, households seek to defend their future welfare against the deterioration of the free resource by increasing their ability to consume private goods in substitution for $R_t$. They can do so by keeping both their saving rates and their labor supply relatively high. This generates perpetual growth, which would not be possible if the households' consumer activities did not have negative effects on

[10] It is also worth noting that the prediction that population increase will raise the rate of growth of per capita output is entirely consistent with the predictions made by models of endogenous technological change (see Grossman and Helpman, 1991; Aghion and Howitt, 1992; Kremer, 1993). In models of technological change an increase in population spurs technological change and economic growth by increasing the size of the market, because the cost of inventing a new technology is independent of the number of people who use it. According to Kuznets (1960) an increase in population boosts technological progress by favouring intellectual contacts among people and labor specialization. In this way, greater population density can explain the disproportionally larger number of innovations in cities. However, our prediction depends on the increase in negative externalities due to congestion (increased pressure on environmental and social assets), rather than on positive externalities due to scale effects.

environmental quality.

Our model has considered a purely 'laissez-faire' economy where decision making is decentralized and markets for some environmental resources are missing. However, the increasing impact on environmental quality of the negative externalities generated in the course of the growth process calls for some collective action (creation of markets for environmental resources, creation of authorities managing these resources…). Indeed, "even for those dimensions of environmental quality where growth seems to have been associated with improving conditions, there is no reason to believe that the process is an automatic one", since "the strongest link between income and pollution in fact is via an induced policy response" (Grossman and Krueger, 1995: pp.371-372).[11] Hence, economic growth is no substitute for environmental policy (see also Arrow et al., 1995). In this connection, our model suggests that an environmental policy which is successful in limiting the negative effects of producer and consumer activities on the environment weakens an important driving force pushing economic growth.

## Appendix

[11] Despite the consensus that at least some pollutants exhibit inverted-U, or 'Kuznets' relationships with per capita income, there is no conclusive evidence on the relationship between economic growth and environmental degradation. For instance, Selden and Song (1994) argue that the evidence showing that carbon dioxide emissions appear to rise monotonically with income supports the conjecture according to which pollutants that are costly to abate and have primarily global (as opposed to own-country) effects do not exhibit inverted-U relationships with income. Arrow et al. (1995: p.92) note that "reductions in one pollutant in one country may involve increases in other pollutants in the same country or transfers of pollutants to other countries". Estimating a dynamic model, De Bruyn et al. (1998) show that economic growth has a direct positive effect on the levels of emissions, thus supporting the radical standpoint, according to which the idea that economic growth can be good for the environment is 'false and pernicious nonsense' (see Ayres, 1995).

*The Pareto-optimal path in the presence of negative externalities*

A benevolent planner would internalize the negative externalities caused by the consumer activities. For simplicity and without loss of generality, we normalize the large number of households to be one. Therefore, maximizing the Hamiltonian

$$H_t = \sum_{i=0}^{\infty} \theta^i \left\{ U_{t+i} - \lambda_{t+i}\left[K_{t+i+1} - AK_{t+i}h_{t+i}^{\alpha} - (1-\sigma)K_{t+i} + C_{t+i}\right] - \eta_{t+i}\left[R_{t+i+1} - R_{t+i} - rR_{t+i}\left(1 - \frac{p_{t+i}}{E}\right)\right]\right\} \text{ with}$$

respect to $C_t$, $h_t$, $K_{t+1}$ and $R_{t+1}$, where $\lambda_{t+i}$ and $\eta_{t+i}$ are the multipliers, we obtain the following conditions that the Pareto-optimal path must satisfy:

$$\delta R_t^{\delta} C_t^{\delta-1} u_{x_t} = \lambda_t + \frac{\eta_t r\rho\gamma\delta R_t^{1+\gamma\delta} C_t^{\gamma\delta-1}}{E}, \tag{A1a}$$

$$-u_{h_t} = A\alpha K_t h_t^{\alpha-1}\lambda_t, \tag{A1b}$$

$$\lambda_t = \lambda_{t+1}\theta\left(Ah_{t+1}^{\alpha} + 1 - \sigma\right), \tag{A1c}$$

$$\eta_t = \eta_{t+1}\theta\left\{1 + r\left[1 - \frac{\rho(1+\gamma\delta)(R_{t+1}C_{t+1})^{\gamma\delta}}{E}\right]\right\}. \tag{A1d}$$

The optimal path must also satisfy the laws of motion (4) and (5b), and the transversality conditions:

$$\lim_{t\to\infty} \theta^t \lambda_t K_t = 0, \tag{A2a}$$

$$\lim_{t\to\infty} \theta^t \eta_t R_t = 0. \tag{A2b}$$

Note that $\eta_t$ captures the increment in the discounted sequence of future utilities that the representative household can obtain thanks to a marginal increase in the current endowment of the free resource: comparing (A1a) with (8a) shows that the benevolent planner also takes account of the negative effect of a marginal increment in private consumption on future environmental quality.

*Example where consumption and leisure are complements ( $u_{h_t x_t} < 0$ )*

Let us assume that

$$u(x_t, h_t) = (x_t - \underline{x})^{\varphi}(1-h_t)^{1-\varphi}, \quad \underline{x} > 0, \; 0 < \varphi < 1, \tag{A3}$$

where $\underline{x}$ is a (constant) subsistence level of consumption.

Given (A3), one can use (4) and (8) to obtain the system of equations that – together with (5a) or (5b) -- governs this economy:

$$\theta(Ah_{t+1}^{\alpha}+1-\sigma)(C_{t+1}R_{t+1})^{\delta}\frac{[(C_{t+1}R_{t+1})^{\delta}-\underline{x}]^{\varphi-1}}{(1-h_{t+1})^{\varphi-1}C_{t+1}}=(C_tR_t)^{\delta}\frac{[(C_tR_t)^{\delta}-\underline{x}]^{\varphi-1}}{(1-h_t)^{\varphi-1}C_t}, \tag{A4a}$$

$$K_{t+1}=AK_th_t^{\alpha}+K_t(1-\sigma)-C_t, \tag{A4b}$$

where $K_t=\dfrac{(1-\varphi)C_t[(C_tR_t)^{\delta}-\underline{x}]}{A\alpha\varphi\delta(C_tR_t)^{\delta}(1-h_t)h_t^{\alpha-1}}$.

Note that equations (A4a) and (A4b) can be rewritten as

$$\frac{(1+\nu_t)(1-h_{t+1})^{\varphi-1}}{\theta(Ah_{t+1}^{\alpha}+1-\sigma)x_{t+1}[x_{t+1}-\underline{x}]^{\varphi-1}}=\frac{(1-h_t)^{\varphi-1}}{x_t[x_t-\underline{x}]^{\varphi-1}},\ \nu_t\equiv\frac{C_{t+1}-C_t}{C_t}, \tag{A5a}$$

$$\frac{(1+\nu_t)(1-\varphi)[x_{t+1}-\underline{x}]}{A\alpha\varphi\delta x_{t+1}(1-h_{t+1})h_{t+1}^{\alpha-1}}=(Ah_t^{\alpha}+1-\sigma)\frac{(1-\varphi)[x_t-\underline{x}]}{A\alpha\varphi\delta x_t(1-h_t)h_t^{\alpha-1}}-1. \tag{A5b}$$

Equations (A5) and (13a) governs the equilibrium path of this economy in the absence of negative externalities. An equilibrium path is a balanced growth path if $\nu_{t+1}=\nu_t=\nu$, $\pi_{t+1}=\pi_t=\pi$ and $h_{t+1}=h_t=h$ in equations (A5) and (13a). It is easy to check that a balanced growth path governed by (A5) and (13a) must have $\nu_{t+1}=\nu_t=\nu=0$.

In the presence of negative externality, one can use (6) to rewrite equation (5b) as

$$\frac{x_{t+1}^{1/\delta}}{(1+\nu_t)}-x_t^{1/\delta}=rx_t^{1/\delta}\left(1-\frac{I\rho x_t^{\gamma}}{E}\right). \tag{A5c}$$

Thus, the system governing the equilibrium path of this economy in the presence of negative externalities consists of three difference equations in $\nu_t$, $x_t$ and $h_t$ ((A5a), (A5b) and (A5c)). Hence, along a balanced growth path, one must have $\nu_{t+1}=\nu_t=\nu$, $x_{t+1}=x_t=x$ $\left(\text{entaling}\,1+\pi=1/(1+\nu)\right)$ and $h_{t+1}=h_t=h$ in (A5). By solving the system (A5) for $\nu_{t+1}=\nu_t=\nu$, $x_{t+1}=x_t=x$ and $h_{t+1}=h_t=h$, one can check that the solving triple $(\nu,x,h)$ is such that in general $\nu\neq0$ ($\nu=0$ only for particular combinations of parameter values).

*Dynamics in the absence of negative externalities*

One can rewrite the system (11) as

$$h_{t+1}=\left\{\left\{K_t(Ah_t^{\alpha}+1-\sigma)-\left[\frac{A\alpha\beta\delta K_t(1-h_t)^{\zeta}}{(1-\beta)h_t^{1-\alpha}R_t^{\delta(\xi-1)}}\right]^{1/[1-\delta(1-\xi)]}\right\}\frac{h_{t+1}^{\alpha-1}(1-h_{t+1})^{\zeta}}{\theta AK_t h_t^{\alpha-1}(1-h_t)^{\zeta}}-\frac{(1-\sigma)}{A}\right\}^{1/\alpha}, \quad \text{(A6a)}$$

$$K_{t+1}=K_t(Ah_t^{\alpha}+1-\sigma)-\left[\frac{A\alpha\beta\delta K_t(1-h_t)^{\zeta}}{(1-\beta)h_t^{1-\alpha}R_t^{\delta(\xi-1)}}\right]^{1/[1-\delta(1-\xi)]}. \quad \text{(A6b)}$$

The combinations of $K_t$ and $h_t$ that satisfy (A6a) and are such that $h_{t+1}$-$h_t$=0 (see this locus in figures 1 and 2) are given by

$$\Gamma(K_t, h_t)=\left\{\left\{K_t(Ah_t^{\alpha}+1-\sigma)-\left[\frac{A\alpha\beta\delta K_t(1-h_t)^{\zeta}}{(1-\beta)h_t^{1-\alpha}R_t^{\delta(\xi-1)}}\right]^{1/[1-\delta(1-\xi)]}\right\}\frac{1}{\theta AK_t}-\frac{(1-\sigma)}{A}\right\}^{1/\alpha}-h_t=0. \quad \text{(A7a)}$$

Similarly, the combinations of $K_t$ and $h_t$ that satisfy (A6b) and are such that $K_{t+1}$-$K_t$=0 (see this locus in figures 1 and 2) are given by

$$\Lambda(K_t, h_t)=K_t(Ah_t^{\alpha}+1-\sigma)-\left[\frac{A\alpha\beta\delta K_t(1-h_t)^{\zeta}}{(1-\beta)h_t^{1-\alpha}R_t^{\delta(\xi-1)}}\right]^{1/[1-\delta(1-\xi)]}-K_t=0. \quad \text{(A7b)}$$

One can check that $\left.\frac{\partial h_t}{\partial K_t}\right|_{\Gamma(.)=0}=-\frac{\frac{\partial\Gamma(.)}{\partial K_t}}{\frac{\partial\Gamma(.)}{\partial h_t}}\begin{Bmatrix}>\\<\end{Bmatrix}0$ whenever $\frac{\partial\Gamma(.)}{\partial K_t}\begin{Bmatrix}<\\>\end{Bmatrix}0$, where $\frac{\partial\Gamma(.)}{\partial K_t}\begin{Bmatrix}<\\>\end{Bmatrix}0$ whenever $\xi\begin{Bmatrix}<\\>\end{Bmatrix}1$, and $\frac{\partial\Gamma(.)}{\partial h_t}>0$. Similarly, $\left.\frac{\partial h_t}{\partial K_t}\right|_{\Lambda(.)=0}=-\frac{\frac{\partial\Lambda(.)}{\partial K_t}}{\frac{\partial\Lambda(.)}{\partial h_t}}\begin{Bmatrix}>\\<\end{Bmatrix}0$ whenever $\frac{\partial\Lambda(.)}{\partial K_t}\begin{Bmatrix}<\\>\end{Bmatrix}0$, where $\frac{\partial\Lambda(.)}{\partial K_t}\begin{Bmatrix}<\\>\end{Bmatrix}0$ whenever $\xi\begin{Bmatrix}<\\>\end{Bmatrix}1$, and $\frac{\partial\Lambda(.)}{\partial h_t}>0$.

**FIGURE 1**

The phase diagram of the economy in the absence of negative externalities ($\xi$>1)

$h_t$

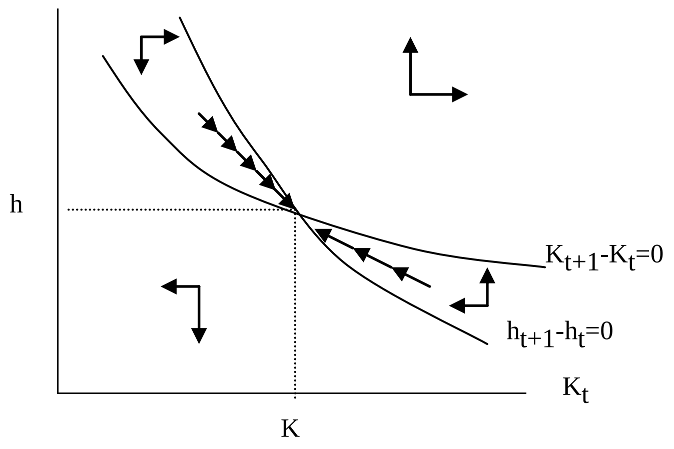
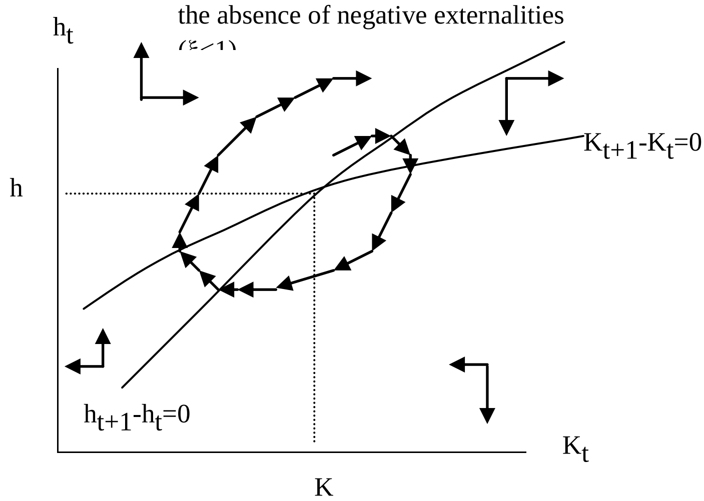



**FIGURE 2**

The phase diagram of the economy in the absence of negative externalities ($\xi<1$)




Note that the economy is saddle-path stable whenever $\xi>1$ and unstable whenever $\xi<1$. Furthermore, note that any trajectory that does not converge to (K,h) violates the constraints $K_t \geq 0$ and $0 \leq h_t \leq 1$ in finite time. Thus, it cannot be an equilibrium path (whenever $\xi<1$, only the trajectory coinciding with (K,h) $\forall$t does not violate the constraints $K_t \geq 0$ and $0 \leq h_t \leq 1$).

Solving (5a) and (11) for $R_{t+1}=R_t=R$, $K_{t+1}=K_t=K$ and $h_{t+1}=h_t=h$, one obtains:

$$R=E, \tag{A8a}$$

$$K=\left[\frac{(1-\beta)(Ah^{\alpha}-\sigma)^{1-\delta(1-\xi)}}{A\alpha\beta\delta h^{\alpha-1}(1-h)^{\zeta}R^{\delta(1-\xi)}}\right]^{1/\delta(1-\xi)}, \tag{A8b}$$

$$h=\left[\frac{1-\theta(1-\sigma)}{\theta A}\right]^{1/\alpha}. \tag{A8c}$$

Linearizing (11) around (A8b) and (A8c) yields the following characteristic equation:

$$\chi^2 - \left\{2-(1-\theta)\left[\frac{(1-\alpha)(1-h)+\zeta h+\dfrac{\alpha(1-h)[1-\theta(1-\sigma)]}{[1-\delta(1-\xi)]}}{(1-\alpha)(1-h)+\zeta h+\alpha(1-h)[1-\theta(1-\sigma)]}\right]\right\}\frac{1}{\theta}\chi+\frac{1}{\theta}=0,$$ where $\chi_1$ and $\chi_2$ are the characteristic roots and h is given by (A8c). As a numerical example, let A=.5, $\alpha=\delta=1$, $\sigma=.2$, $\theta=.8$, $\zeta=.1$, $\beta=.5$, E=4 and r=.5. Given these parameter values, $R_t$ converges monotonically to R=4 and h=.9. Setting $\xi=2$, one obtains: K=1.5886565, $\chi_1=.9059554$ and $\chi_2=1.3797588$ (the system obtained by linearizing (11) around (h=.9, K=1.5886565) exhibits saddle-path stability). Setting $\xi=.9$, one obtains: K=.0097656, $\chi_1=1.0391$ and $\chi_2=1.2029594$ (the system obtained by linearizing (11) around (h=.9, K=.0097656) is unstable).

*Saddle-path stability of the system obtained by linearizing ((12) and (13c) around the steady-state values of $\mu_t$, $Z_t$ and $h_t$*

By using (12a), one can rewrite (12b) and (13c) as a system of difference equations in $h_t$ and $Z_t$:

$$\Phi(h_{t+1},h_t,Z_t)=\frac{\theta\left(Ah_{t+1}^{\alpha}+1-\sigma\right)}{h_{t+1}^{\alpha-1}(1-h_{t+1})^{\zeta}}-\frac{\left(Ah_t^{\alpha}+1-\sigma\right)}{h_t^{\alpha-1}(1-h_t)^{\zeta}}+\left\{\frac{A\alpha\beta\delta\left[Z_t h_t^{\alpha-1}(1-h_t)^{\zeta}\right]^{\delta(1-\xi)}}{(1-\beta)}\right\}^{\frac{1}{[1-\delta(1-\xi)]}}=0, \quad \text{(A9a)}$$

$$\Psi(h_{t+1},Z_{t+1},h_t,Z_t)=\frac{Z_{t+1}(1-h_{t+1})^{\zeta}}{\theta(Ah_{t+1}^{\alpha}+1-\sigma)h_{t+1}^{1-\alpha}}-\frac{Z_t h_t^{\alpha-1}}{(1-h_t)^{-\zeta}}\left\{1+r-\frac{rI\rho}{E}\left[\frac{A\alpha\beta\delta Z_t h_t^{\alpha-1}}{(1-\beta)(1-h_t)^{-\zeta}}\right]^{\frac{\gamma\delta}{[1-\delta(1-\xi)]}}\right\}. \quad \text{(A9b)}$$

Linearizing (A9) around a pair (h, Z) satisfying (14b) and (14c) yields the following characteristic equation:

$$\chi^2-\left[\frac{\Phi_{Z_t}\Psi_{h_{t+1}}-\Phi_{h_t}\Psi_{Z_{t+1}}-\Phi_{t+1}\Psi_{Z_t}}{\Phi_{h_{t+1}}\Psi_{Z_{t+1}}}\right]\chi-\left[\frac{\Phi_{Z_t}\Psi_{h_t}-\Phi_{h_t}\Psi_{Z_t}}{\Phi_{h_{t+1}}\Psi_{Z_{t+1}}}\right]=0,$$ where $\chi_1$ and $\chi_2$ are the characteristic roots and all the derivatives are evaluated at $h_{t+1}=h_t=h$ and $Z_{t+1}=Z_t=Z$. As a numerical example, let $\beta=r=\sigma=.5$, $A=\alpha=\delta=\zeta=\rho=\gamma=I=1$, $\theta=E=.8461538$ and $\xi=1.01$. Given these parameter values, one has: h=.8, Z=5, $\mu=.1$, $\chi_1=.35779$ and $\chi_2=1.1773$ (the system obtained by linearizing (A9) around (h=.8, Z=5) exhibits saddle-path stability).

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