

# Genesis and evolution of the multiplier-accelerator model in the years of high theory<sup>§</sup>

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## Abstract:

In the 1930s there was a rapid convergence of the most important schools of thought in macrodynamics, in particular those rooted in the Marxian, Wicksellian, Marshallian, and American traditions, towards an approach to dynamics based on the interaction between the multiplier and the accelerator principle. This process culminated at the end of the period in the publication of a very influential prototype analytical model describing self-sustaining business cycles (Samuelson 1939a,b), and in the contemporaneous publication of a no less influential, although partially misunderstood, semi-analytic theoretical framework of cyclical growth (Harrod 1939). These two contributions were to initiate Keynesian macrodynamics, articulated in the two separate branches of business cycle and growth theory that dominated the economic literature for many decades. We explain this convergent genetic process mainly on the basis of two factors that proved to be mutually synergic. First of all we have to consider the birth of quantitative dynamics based on the systematic application of functional equations and of econometric methods to economic dynamics. Second, we have to consider the influence of the Great Contraction of the 1930s that prompted the economists to provide simple and quantifiable recipes to get out of the crisis and control the economy. The results of this evolutionary process cannot be read exclusively in terms of progress. We try to identify the steps forward and the losses brought about by this process that originated Keynesian macrodynamics

*Key Words:* Accelerator principle, Multiplier, Dynamic multiplier, Interaction between multiplier and accelerator principle, Great Contraction, Great Depression, Endogenous business cycles, Disequilibrium dynamics, Instability

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## 1. Introduction

We may say that the year 1939 is the date of birth of Keynesian macrodynamics. The happy event is marked by the birth of *twin models*: a prototype analytic model of the business cycle (Samuelson 1939a,b), and a prototype conceptualization of macroeconomic dynamics including a seminal model of growth (Harrod 1939). The parents of both models were the dynamic version of the multiplier and the acceleration principle. The pregnancy of their interaction emerged only in the late 1930s after a few decades of separate life. The first economist who foreshadowed it was probably Harrod in 1933 soon followed by Ohlin in 1934. A first detailed analysis of its consequences came only with Harrod (1936) and, independently, Lundberg (1937). However, a rigorous analysis of the effects of the interaction emerged only with the prototype models mentioned above. The family of business cycle models springing from Samuelson's prototype model was complemented by a theory, at least in the weak sense of a "set of instructions for building models" (Lucas 1981) of similar, or at least related, inspiration. At the same time, Harrod's theory of dynamics bifurcated in the hands of subsequent growth theorists in a family of analytic models of growth and a 'theory' of growth in the same weak sense mentioned above. By contrast Harrod's theory of business cycles in a growing economy was misunderstood and then neglected, notwithstanding his lifelong efforts to clarify his original research programme.

The interaction between multiplier and accelerator principle has given common foundations to the models analysing Keynesian macrodynamics, in both its branches of business cycle and growth analysis, for a very long period from the late 1930s up to the late 1950s and beyond. Although the consistency of this family of multiplier-accelerator (henceforth MA) models with Keynes's own ideas has been often questioned, they were adopted by many mainstream Keynesian economists in the 1940s, 1950s and 1960s as the dynamic counterpart of the IS-LM static framework, capable of extending the reach of Keynesian macroeconomics to both business cycle and growth issues. In this period, alternative approaches to macrodynamics were suggested and pursued (in particular by Frisch 1933 and Slutsky 1937) but only the equilibrium approach promoted by the New Classical Economists in the early 1970s succeeded in demoting the MA models from the pre-eminent position occupied in the previous thirty years (see in particular Lucas

1981 and Lucas & Sargent 1981 for early collections of the most influential papers that led to the anti-Keynesian counter-revolution).

The two basic ingredients of the model (the multiplier and the accelerator principle) have a much longer pedigree. However, the previous analysis of both the multiplier and the accelerator principle had been mainly qualitative or improperly modelled. Samuelson's 1939 contributions may be considered as the first rigorous model able to specify the conditions under which the interaction between the multiplier and the accelerator principle may explain economic fluctuations. We thus take this model as a prototype for the subsequent literature in the sense that it set the language and the approach dominating the debate on business cycles and related issues for decades. Analogously, the semi-analytical conceptual framework worked out by Harrod in 1939 provided the basic concepts of Keynesian growth theory, in particular the crucial distinction between effective, warranted, and natural rates of growth providing also their first elementary formalization.

In this paper we analyze the proximate genesis of these influential models in the 1930s. The first steps of this genetic process could be retro-dated by many decades as we will hint in our analysis. We claim, however, that a key factor in the convergence, coagulation and distillation of different traditions of thought towards the prototype Keynesian models of business cycles and growth was the Great Contraction of the 1930s. The latter produced a radical turn in economics, in particular in the field that in the late 1930s, under the influence of Keynes, started to be called macroeconomics. This is why these years came to be called the 'years of high theory' (Shackle 1967) and rightly so: the huge anomalies in received doctrine, made altogether evident by the crisis, required a revision of the basic assumptions of economic theory.<sup>1</sup>

The genetic process reconstructed in this paper was characterized by fairly rapid innovations. The period starts with the publication by Keynes of the *Treatise on Money* (1930), an ambitious book in which Keynes tried to build up a creative synthesis

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<sup>1</sup> Shackle took 1926 as starting year of his analysis. This was appropriate in a general study for at least two reasons. It allowed a clearer understanding of the radical turn in economic theory by comparing the state of the art in the late 1920s with that of the 1930s. However, the main innovations in the late 1920s emerged in microeconomic theory and were independent of the crisis. In particular the 'theory of imperfect competition' put forward by Chamberlain and Robinson reflected the growing importance of big oligopolistic firms in the expansion phase preceding the Great Contraction. In our case the choice of 1930 as starting point of the analysis allows a more focused examination of the evolution of macrodynamics without crucial losses.

between the two most prestigious schools of macroeconomics rooted in the Marshallian and Wicksellian traditions. In such attempted synthesis neither the multiplier nor the accelerator principle played any role, while these two pieces of analysis occupied centre stage in the immediately subsequent years and were soon combined to bring about the prototype model of Keynesian macrodynamics. The interaction of the multiplier and the accelerator principle in business-cycle theory was first examined in the English language by Harrod (1936) and then first rigorously formalized, under the influence of his mentor Alvin H. Hansen, by Samuelson (1939a,b). Before him, Kalecki (1933/1990; 1935a,b; 1939, Ch. 6), in his turn strongly influenced from the point of view of the mathematical techniques employed by the Tinbergen's (1931) work on the shipbuilding cycle, had already produced what can be perhaps considered the first mathematical model of the business cycle referred to the entire economy. Although, strictly speaking, Kalecki did not use the accelerator principle to explain investment in fixed capital, his model shows many similarities with subsequent MA models. As for growth theory, the prototype conceptualization may be found in the seminal paper in which Harrod (1939) suggested new general foundations for macrodynamics, i.e. in his view both business cycles and what is now called growth theory.

We explain this revolutionary change mainly on the basis of two factors that proved to be mutually synergic. First of all we have to consider the birth of quantitative dynamics based on the application of functional equations and econometric methods to economic dynamics. Frisch founded the Econometric Society in 1930, while Tinbergen (1931) provided the first example of mathematical theory of business cycles in reference to the sector of ships construction. Second, as mentioned before, we have to consider the influence of the Great Contraction that prompted the economists to provide simple and quantifiable recipes to get out of the crisis and control the economy. The result of this evolutionary process, as often happens, cannot be read exclusively in terms of progress. It produced benefits but also losses and we will try to identify both of them.

The structure of the paper is as follows. In section 2 we introduce and discuss the basic 'ingredients' of the MA models: the multiplier, the accelerator principle, and the basics of their interaction. In section 3 we discuss the Marxian background of macrodynamics with special emphasis on the influence of Kalecki on the gestation of Keynesian macrodynamics. In section 4 we analyze the influence of the Wicksellian

tradition and in particular the pioneering role of Ohlin and Lundberg, while in section 5 we analyze the influence of the Marshallian tradition and in particular the crucial impact of Harrod on the emergence of the MA interaction models. In section 6 we examine the influence of the American tradition and in particular the impact of Hansen on the immediate genesis of the prototype model of business cycles. Conclusions follow.

## **2. The interaction between the accelerator principle and the multiplier: vision and analysis**

The debate on the origins of the multiplier and the accelerator principle has a long history and leaves somehow open the question of who was really ‘the first’. This is hardly surprising given the importance that the two mechanisms had in the development and establishment of Keynesian macrodynamics.<sup>2</sup>

In order to understand its genetic process and assess the contribution given by different traditions of thought and single eminent economists we have to consider not only the analytical steps but also their underlying vision. We need thus a preliminary examination of the standard textbook version of the two basic pieces of analysis, the multiplier and the accelerator principle, in the light of their underlying vision. At the end of this section we combine the two basic pieces in a unified basic MA model, considered in its turn in its standard textbook version, to understand the problems raised by the clash of the two underlying visions that explains the late emergence of their synthesis and is at the root of different interpretations and developments. To keep an intuitive understanding of the semantic and pragmatic implications of these theoretical constructs we will make use of representations in terms of block-diagrams.

The ‘fabrication’ of the MA model requires a vision of the circular flow of income and expenditure and an analytical apparatus capable of tracing the dynamic path of the economy under the influence of these two mechanisms. All the inputs required by the ‘fabrication’ of the MA model were at the disposal of the economists much before the

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<sup>2</sup> See the recent discussion on the pages of *History of Political Economy*, initiated with an article by Heertje and Heemeijer (2002) and continued with a debate involving, in chronological order, Samuelson (2002), Siven (2003), Besomi (2003), Heertje (2003) and Samuelson (2003) again. More recently, the debate was joined by Fiorito (2006, 2007). The arguments at stake are better understood by reading in addition Samuelson (1939a,b; 1959; 1988), Harrod (1936; 1939) and Hansen (1937; 1938; 1941).

period of our analysis (the 1930s). A quite sophisticated model of the circular flow, the *Tableaux économique*, had been already drafted by Quesnay in 1758 and was updated to modern capitalist relations and further developed by Marx, mainly through the reproduction schemas developed in the second volume of the *Capital* (1885/1956). The multiplier that was implicit in this vision had been already made explicit by Mummery & Hobson (1889) and a few other authors before the timely and influential version of Kahn (1931). This version was soon clarified and extended by Warming (1932) and developed as mean of determination of income by Keynes (1936). As for the accelerator, it is an obvious consequence of a technological requirement that is crucial in a capitalist economy. Its analytic formulation in terms of a relation between the variation of income and the induced variation of investment had been already made explicit by Aftalion (1909, 1913), Hawtrey (1913) and Clark (1917). All these ingredients were put together and synthesized in a coherent framework only in the late 1930s by Harrod (1936, 1939), Lundberg (1937) and Samuelson (1939a,b). Why was it so difficult to put the pieces together? Why this happened only in the late 1930s? Why when eventually an analytical synthesis emerged, it was so successful?

The vision underlying the multiplier is based on the crucial role of money in a capitalist economy to propel the circulation of goods, services and productive factors. As it was brilliantly synthesized by Clower (1967, p. 5), in a monetary economy “money buys goods, goods buy money but goods do not buy goods”. This elementary but deep and pervasive principle that distinguishes a monetary economy from a barter economy, translates from the point of view of transactors in the circular flow of income and expenditure. The expenditure in goods and services has to be financed by income received in advance by the buyers and creates an equivalent income for the sellers of these goods and services; this income received by sellers finances their expenditure, and so on. The multiplier derives very simply from a thorough understanding of this cash-in-advance constraint in the circular flow of expenditure and income (see section 3.1). More precisely, in the light of this constraint, what impact on aggregate income or employment will eventually have an injection of a certain amount of exogenous expenditure in the circular flow? As is obvious, the answer to this question is quite important from the point of view of policy, particularly when persistent structural unemployment plagues the economy. Generally speaking the answer to this question

depends on two crucial conditions. First, it depends on the level and nature of unemployment. Whenever the economy is characterized by full employment in labour and other productive factors, the effect can only be monetary in the form of accelerating inflation, while on the contrary the effects on the real economy may be beneficial to the extent that there are idle productive factors and resources. We may immediately understand why the interest for some version of the multiplier has always emerged in periods characterized by persistent structural unemployment brought about by insufficient effective demand. This observation clarifies in particular why the attention for the multiplier flourished in the 1930s in consequence of the Great Contraction and of the ensuing persistent structural unemployment. This also explains why the usual multiplier model is fixprice as it assumes implicitly that the economy is far enough from full employment equilibrium to justify such assumption and to justify the use of the multiplier. Under these assumptions the second question arises. Which is the relationship between the increment of exogenous expenditure and the ensuing increment of aggregate income? The answer depends on the 'leakages' characterising the circular flow that, excluding the role of prices, depend in particular on money hoarding, saving, taxation, or import of foreign goods and services. In the absence of leakages, the process of increase of economic flows triggered by the exogenous additional expenditure would go on without limit. In the ancient economies a significant leakage was money hoarding. In modern economies there are two other crucial leakages. In a country assumed to be closed to foreign trade, a significant part of the income is saved. In addition, in a country open to foreign trade a significant part of the additional purchasing power is expended in imported goods and services. Of course the higher the leakages, the lower the value of the multiplier as it converges more rapidly towards zero by losing much of its strength at each round of circuit. We can represent the circular flow of expenditure and income through a block diagram that may help an intuitive understanding of the underlying mechanism (see fig.1).<sup>3</sup>

INSERT FIG. 1

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<sup>3</sup> To the best of our knowledge this representation was first used for economic systems by Phillips (1954) and then made popular among economists by Allen (1967) and Lange (1970).

In this block-diagram  $E'$  represents the additional exogenous expenditure injected in the economy that added to the endogenous expenditure  $E^*$  determines the overall aggregate expenditure  $E$  that translates in an equivalent value of aggregate income  $Y$  received by the sellers of goods and services. In other words:

$$E = E' + E^*$$

$$E^* = kY$$

$$Y = E$$

from which we derive immediately that the increase in aggregate income is a multiple of the increased flow of aggregate exogenous expenditure:

$$\Delta Y = \frac{1}{1-k} \Delta E'$$

where  $1/(1-k)$  is the multiplier. The crucial point is that the increase in endogenous aggregate expenditure in the first lap of the circuit is less than the increase in aggregate income because  $0 < k < 1$  while  $1-k$  represents the leakage in the circuit, and so on at each lap of the circuit. This is, more or less, what the economists had in mind when discussing to what extent a new programme of public works, or an increased budgetary deficit, could be beneficial to employment and aggregate income  $Y$ . Kahn (1931) was the first to provide a clear analytic measure of the effects of the multiplier by using the well known formula of converging series. It is interesting to observe that this analytic result is obtained by mathematics on the basis of a recursive computational process that mimics in formal terms the repetition of the circuit. Kahn's was an employment multiplier, but we can easily derive an employment multiplier from an expenditure multiplier provided that we know the technological coefficients of a certain economy.

Kahn was imprecise in the identification of the relevant leakages stressing mainly the role of imported goods. Soon Warming (1932) clarified that in a closed economy the main leakage is saving. Keynes immediately adopted Warming's version of the multiplier using it to determine in the short run the aggregate income of a closed economy by assuming that all the investment is exogenous and that the multiplier is an instantaneous, or very quick, process (see fig. 1). So doing he made a crucial step forward towards a prototype MA model that describes the dynamic behaviour of the aggregate income of a closed economy. However, in order to focus on the short period, he adopted an instantaneous and static version of the multiplier that is altogether

inconsistent with the accelerator as it presupposes a dynamic process of determination of income and the adaptation of the capital stock to this dynamic process.

As is well known, the accelerator explains the endogenous investment as induced by the need of fitting the stock of capital  $K_t$  to the growth of aggregate income given a ratio  $v$  that measures the desired ratio between aggregate income and the stock of capital utilized to produce this income:

$$I_t^* = v(Y_t - Y_{t-1})$$

where the endogenous investment  $I_t^* = K_t - K_{t-1}$  and  $v$  indicates the capital-output ratio.

At first sight the accelerator is an obvious way to extend the multiplier to dynamic analysis beyond the short period because it accounts for endogenous investment and for the adaptation of capital stock to the dynamics of economic activity (see fig. 2).

#### INSERT FIG. 2

What has to be explained is why this natural marriage took so long to coagulate. We believe that one reason is that it implies in general a systematic imbalance between investment and saving that the economists before the Great Contraction were unprepared to accept. Keynes prepared the terrain by emphasising that the investment brings about saving via income; however Keynes himself in the *General Theory* insisted that investment and saving (*ex post*) were identically equal for accounting reasons. The other source of confusion is that the feedback between income and expenditure via consumption (the multiplier) is between the level of income and the level of consumption while the feedback via endogenous investment is between a variation of income and the level of induced investment. This asymmetry was found confusing and difficult to handle by economists not yet properly trained in dynamic functional equations.

These preliminary observations should be kept in mind while we struggle to reconstruct the process of fabrication of the prototype MA models in the 1930.

### 3. Marx and Kalecki

#### 3.1. *The Marxian connection*

Marx is widely recognized as the father of modern macrodynamics. The strict link between the reproduction schemas developed in the second volume of the *Capital* and modern growth theory has been often pointed out. As for business cycles theory, it is generally recognized that Marx's achievements, though dispersed in different writings in a non-systematic way, "assure him high rank among the fathers of modern cycle research" (Schumpeter 1942, p. 51). No wonder that we find in the literature claims of Marx's priority also on the use of the multiplier and the accelerator principle in macrodynamic analysis. The link with the multiplier is seen in the reproduction schemas (see for a useful survey and discussion Trigg 2006), while hints on the accelerator principle are found in the third volume of the *Capital* and in the *Theories of Surplus Value* (see, e.g. Junankar 1982, 1987). A thorough assessment of this issue would require a careful and lengthy analysis of all the writings of Marx. Taking account of the purposes of this paper that focuses on the genesis of formalized MA models we will limit ourselves to consider two crucial pieces of relevant semi-formalized analysis in the *Capital*: the reproduction schemas that are the main reference of the literature on the Marxian anticipation of the 'multiplier' and the Harrod-Domar growth model, as well as the formulas of circulation of capital introduced at the very outset of the first volume of the *Capital* whose bearing on the foundations of multiplier analysis, to the best of our knowledge, has never been pointed out (with the exception of Sordi & Vercelli 2006). The derivations of the multiplier obtained in this section do not justify the claim that Marx was the real originator of the multiplier. The structural approach of Marx is much richer than that of the multiplier so that we should not be surprised that we may derive it from Marxian theoretical constructs through elementary steps that simplify and make explicit similar relations. These derivations build a bridge between Marx's approach and the MA approach that may establish interesting analogies but also measures the distance between the two polarities. In addition derivations of this sort may clarify the meaning and implications of the multiplier and MA models.

A thorough assessment of the Marx's influence on this literature has to distinguish, following Schumpeter, between vision and analysis. This is quite clear if we focus on

the multiplier. There are many derivations that start from Marx reproduction schemas and obtain, after a limited series of algebraic steps, formulas similar to the Kahn-Keynes multiplier (see Trigg 2006). In our opinion, if we examine these derivations in their analytical detail, they show more the distance between the Marxian and Keynesian theory than their affinity. The most interesting convergences may be found from the point of view of the vision of the structural characteristics of the capitalist process of circulation. To clarify this point we derive directly a version of the multiplier from a crucial feature of Marx's vision as expressed at the very beginning of vol. 1 of the *Capital* (1867/1954). As is well known, the first two sections of the *Capital* sketch the genesis of the capitalist mode of production and circulation of commodities reconstructed from the conceptual and historical points of view intertwined in a chemical synthesis (Schumpeter 1954). The exposition ends by the statement of the general formula for capital in the sphere of circulation: M–C–M', that is money-commodity-money, where M' > M (Marx 1867/1954, p. 155). This formula is expressed from the point of view of capital, not of the people involved in the transactions, consistently with the fetishism of capitalist circulation. However, if we take this formula from the point of view of transactors and we ignore the requirement of a surplus, the necessary alternation of purchase (M–C) and sale (C–M) translates into the necessary alternation of expenditure  $e_{ij}$  of agent  $i$  on a given commodity  $j$  and income  $y_{ji}$  of the agent  $j$  that sells the commodity to him so that we get:<sup>4</sup>

$$y_{jit} = e_{ijt}$$

for each transaction where  $t$  is the instant in which the transaction occurs.

By summing all flows of expenditure carried on by the transactor  $i$  in period  $t$  (assumed to be short in the sense of Keynes) and all the flows of income earned by his sellers, we get:

$$e_{it} = \sum_{j=1}^m e_{ijt}$$

and

$$y_{jt} = \sum_{i=1}^n y_{jit}$$

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<sup>4</sup> The analysis that follows draws on and clarifies the discussion in Sordi & Vercelli (2006, pp. 417-418).

Moreover, we have to distinguish between endogenous ( $e_{it}^*$ ) and exogenous ( $e'_{it}$ ) acts of expenditure in relation to a well-defined circuit of monetary flows such that:

$$e_{it} = e_{it}^* + e'_{it}$$

The endogenous act of expenditure is financed by income earned by the buyer in preceding stages of the circuit, while an exogenous act of expenditure is financed by a subject that does not belong to the circuit under examination. In what follows, for the sake of a meaningful comparison with Keynes's multiplier, we assume that the circuit refers to the entire private sector of a given closed economy. Therefore, the exogenous acts of expenditure are those financed by public and foreign agents.

The crucial point is that monetary circulation introduces a possible gap between the earning of money and its expenditure so that generally speaking  $y_{jt}$  does not translate in an equal amount of expenditure  $e_{jt}$ . The money earned may be, at least in part, hoarded or saved rather than immediately used for a new transaction. This introduces a leakage in the circuit of the following type:

$$e_{jt}^* = ky_{jt} \quad 0 < k < 1$$

where  $k$  is the coefficient of leakage.

Summing up all the flows of expenditure and income in the period  $t$  for all the transactors, we obtain

$$Y_t = E_t^* + E'_t = E_t \tag{1}$$

where  $E_t = \sum_{k=1}^{n+m} e_{kt}$ ,  $E_t^* = \sum_{k=1}^{n+m} e_{kt}^*$ ,  $E'_t = \sum_k e'_{it}$  and  $Y_t = \sum_{k=1}^{n+m} y_{kt}$ . The endogenous aggregate

expenditure is a function of the aggregate income taking account of the coefficient of leakage  $k$ :

$$E_t^* = kY_t \tag{2}$$

The elementary macroeconomic system described by the relations (1) and (2) may be summarized as follows:

$$Y_t = kY_t + E'_t$$

from which we derive immediately the multiplier:

$$Y_t = \frac{E'_t}{1-k}$$

This derivation of the Keynesian multiplier from Marx's general formula of capital is meant to clarify to what extent the multiplier shares common assumptions with Marx theory of capital circulation. What is common is above all the acknowledgment of the constraints posed by a monetary economy. First, there is a common assumption of the necessary alternation of purchase and sale that from the point of view of agents implies the necessary alternation of income and expenditure. Second, as Marx emphasizes, the necessary mediation of money in principle "separates purchase and sale" in time and space. This implies that also income and expenditure are similarly separated from the temporal and spatial points of view determining the presence of leakages in the income-expenditure circuit. The analogy is over-emphasized by the fact that in this derivation, as in the multiplier, the values are expressed in prices and the latter are considered constant. In both cases this also implies that the economy is assumed to be sufficiently far from full employment equilibrium to neglect the impact of the multiplier on money prices. On the other hand we have introduced the distinction between endogenous and exogenous expenditure since the multiplier acts on the exogenous expenditure seen as instrument of control of the level of economic activity. This assumption does not contradict the tenets of Marx's approach but was not a main concern for a revolutionary outsider who wanted to change the system rather than controlling it. Finally in our derivation we had to assume a short period horizon as in Keynes, although such an assumption was certainly extraneous to Marx's analysis. For the same reason we also ignored the surplus value and accumulation of capital.

This point may be better clarified with reference to the reproduction schemas discussed by Marx in the second volume of the *Capital* (1885/1956). The derivations of formulas similar to the multiplier are based on Marx's reproduction schemas. We discuss an elementary derivation based on the simple reproduction schema to maximize also in this case the affinity with Marx's multiplier. As is well known, the simple reproduction schema may be written in the following way:

$$O_1 = K_1 + V_1 + S_1$$

$$O_2 = K_2 + V_2 + S_2$$

where the suffix 1 stands for the sector of capital goods and the suffix 2 for the sector of consumption goods.  $O$  stands for output,  $K$  for constant capital,  $V$  for variable capital, and  $S$  for surplus. To derive the multiplier we have to insert the system in the process of

circulation assuming that all the output is realized so that the aggregate expenditure brings about equal income. Consumption depends on income and capital expenditure  $K$  is assumed to be exogenous. Defining  $Y = O - K$  as aggregate net income and  $C$  as aggregate consumption we obtain the following system

$$Y = E$$

$$E = K + C$$

$$C = (1 - \lambda)Y$$

where  $\lambda = (1 - c_1)\sigma$ ,  $c_1$  is the propensity to consume of capitalists and  $\sigma = S/Y$  the share of surplus on income. From this we get a sort of multiplier:

$$Y = \frac{K}{\lambda}$$

Here the leakage is given by the savings of capitalists given by  $\lambda Y$  because it is assumed that the variable capital  $V$  translates integrally in consumption. The multiplier depends thus on the consumption propensity of capitalists  $c_1$  and on the share of surplus on net income. The greater the saving propensity of capitalists and/or the share of surplus on income, the greater the leakage.

This kind of derivation is in our opinion quite misleading. In the reproduction schemas the relations between the variables are equilibrium conditions and not behavioural relations as in the multiplier and in the MA model (although we could interpret the assumptions on consumption as such). This implies that capital expenditure is not exogenous in this context as its volume is fixed by the equilibrium conditions. The simple reproduction schema implies that in the equilibrium represented by the schema the leakage due to a consumption propensity of capitalists lower than one, that is saving, is perfectly compensated by capital expenditures made by the same capitalists. That is why the simple reproduction schema can go on for ever. This clarifies a crucial point for the Keynesian multiplier. Within its static assumptions all the investment has to be exogenous in order to avoid the problem emphasized above. The introduction of induced investment requires a dynamic model so that equilibrium becomes a remote possibility rather than the rule. This is done in the spirit of the Marxian tradition by Kalecki.

### 3.2 Kalecki

In the 1930s, in consequence of the Great Contraction, Marxian economics became much more fashionable than before. Even in the academic world, and even in the departments of economics up to then quite impenetrable to any significant influence from Marxian economics, there was a remarkable diffusion of study groups that drew participants not only from students but also from some teachers and researchers. We may speculate in the light of our observations in the preceding section, that the increasing influence of Marx's vision may have contributed to a climate of opinion favourable to the development of the multiplier and eventually to the emergence of the MA model. From the point of view of economic analysis, the crucial link between the Marxian tradition and the fabrication of the MA model in the years of high theory was represented by Kalecki. He freely reformulated the reproduction schemas of Marx in the light of the contributions of Grossman and Rosa Luxemburg (both Polish) and Tugan Baranowski with an approach heavily influenced by his engineering background oriented to the control of complex systems. This view pushed him to identify the main relations of the circulation of capital as behavioural relations expressing the decision of capitalists while labour is seen as merely passive. The crucial driving force of economic dynamics is thus seen in the feedback between accumulation of capital and profits. This approach requires a clear distinction between endogenous and exogenous variables and this leads him to formulate in explicit form his own version of the multiplier (although this name does not appear in his text) as crucial founding block of his analysis. At the beginning of section 2, entitled *Assumptions* of his *Essay on the Business Cycle Theory* (Kalecki 1933/1990), he expresses the following three equations (with notation slightly modified to facilitate the comparison with other authors):

$$P = C_c + A \quad (3)$$

where  $P$  stands for gross real profits,  $C_c$  for consumption by capitalists, and  $A$  gross accumulation that is "all goods which are used in the reproduction and expansion of fixed capital as well as the increase in inventories" (ibid., p. 69).

The personal consumption of capitalists  $C_c$  is the sum of their exogenous consumption  $C'_c$  and their endogenous consumption  $C_c^*$ :

$$C_c = C'_c + C_c^* \quad (4)$$

The latter is determined by their profits:

$$C_c^* = \lambda P \quad 0 < \lambda < 1 \quad (5)$$

where  $\lambda$  expresses the consumption propensity of capitalists.

Kalecki derives from equations (4)-(5) the following multiplier

$$P = \frac{C'_c + A}{1 - \lambda} \quad (6)$$

The main difference from the Keynesian multiplier is obvious: Kalecki's multiplier is meant to determine profits not income, although the latter may be easily derived from profits as soon as we take account of the share of aggregate profits over aggregate income. On the other hand, the analogy with Kahn-Keynes multiplier is much deeper than in the case of Marx's multiplier. It is derived from behavioural functions (not equilibrium conditions) and 'multiplies' the exogenous expenditure that is clearly identified as such. It is thus fit to be used for control purposes. It expresses in a stylized way the fact that in a 'pure' capitalist economy the control is in the hands of capitalists while workers play only a passive role. This is emphasized in its extreme form through the assumption that workers do not save and do not have 'capitalists' incomes (Kalecki 1933/1990, p. 69). Under these assumptions he may assert the celebrated 'Kalecki principle': the capitalists earn what they spend, while workers spend what they earn, that clearly epitomizes the asymmetry of power determined by the private property of means of production. This is clarified by Kalecki (e.g., Kalecki 1968; see Sardoni 1989) by referring to a somewhat modified version of Marx's schemes of reproduction with three Departments, the first producing investment goods, the second consumer goods for capitalists and the third wage goods (Kalecki 1968, p. 71):

<b>1</b>	<b>2</b>	<b>3</b>	
$P_1$	$P_2$	$P_3$	$P$
$W_1$	$W_2$	$W_3$	$W$
$A$	$C_c$	$W$	$Y$

We thus have:

$$P_1 = A - W_1$$

$$P_2 = C_c - W_2$$

$$P_3 = W - W_3 = W_1 + W_2$$

from which:

$$P = P_1 + P_2 + P_3 = A + C_c$$

This clarifies the meaning of equation (3), that is to say that capitalists' expenditure decisions determine profits and not vice versa.

This asymmetry is partially clouded in the usual version of the Kahn-Keynes multiplier that conflates capitalists' and workers' consumption. However, both versions insist on the crucial point that saving is not the 'cause' of investment but the other way around. Analogies and differences with Kahn-Keynes and Marx's multipliers can be further clarified by making explicit the multiplier between exogenous expenditure and income implicit in the Kalecki's approach. Profits may be expressed as follows:

$$P = \sigma Y$$

where  $\sigma$  represents the share of profits over aggregate income  $Y$ . The endogenous consumption of capitalists can thus be expressed as

$$C_c^* = \lambda \sigma Y$$

so that (6) becomes:

$$Y = \frac{C_c' + A}{\sigma(1 - \lambda)}$$

This clarifies that the income multiplier crucially depends, besides capitalists' consumption propensity, also on the functional distribution of income. Keynes is fully aware of the importance of distribution of income in the multiplier but his observations on this issue did not enter in the standard Keynesian tradition. On the contrary this point shows a clear analogy with Marx's point of view.

With Kalecki the Marxian tradition converges towards the MA model for at least two basic reasons: the multiplier plays a central role in economic dynamics and this role is expressed in terms of dynamic equations. However, the second ingredient of the model, the accelerator, is missing. According to Kalecki (1933/1990), whenever an investment is made, three stages must be discerned each of which takes place at a different point in time: (1) investment orders, (2) production of capital goods, the volume of which per unit of time is equal to the gross accumulation  $A$ , and (3) deliveries of finished industrial equipment. The crucial element in his theory is the specification of the investment orders ( $I$ ). In Kalecki (1933/1990) they are conceived mainly as a positive function of 'gross profitability'  $P/K$  (and a negative relation of the rate of interest); in

any case it is not a function of income variation. This relation is then simplified as a linear function of the exogenous expenditure and the stock of capital:

$$I = m(C'_c + A) - n \quad m > 0, n > 0 \quad (1)$$

This relation may be seen as a further step towards the MA model because it brings in the capital stock in the investment function but it is in stark contrast with the accelerator. Kalecki explains why: “an increase in the production of consumer goods can take place without any expansion of capital equipment, merely through increasing its employment” (ibid., p. 105). He therefore criticizes explicitly Aftalion’s theory on this point because it “is based on the same false assumption of constancy of capital employment which runs through all Aftalion’s theory” (ibid., p. 105).<sup>5</sup>

There is no doubt, however, that with Kalecki the Marxian tradition converges in a substantial way towards the MA model. As Joan Robinson (1964, p. 95) observed commenting on the disputed question of the priority between Kalecki and Keynes, “the interesting thing is that two thinkers, from completely different political and intellectual starting points, should come to the same conclusion”.

## 4. The Wicksellian tradition

### 4.1. *The Wicksell connection*

The study of the dynamics of aggregate economic phenomena has been pursued in the 18<sup>th</sup> and 19<sup>th</sup> Centuries mainly on the basis of the equation of exchange expressing the relationship between the quantity of money in circulation multiplied by its velocity of circulation and the level of a general index of the price level multiplied by an index of the aggregate quantity of real goods traded. The prevailing causal interpretation of the basic identity was provided by the quantitative theory of money that assumed the exogeneity of money supply and a substantial stability of its velocity of circulation suggesting a monetary explanation of inflation and the independence of real dynamics

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<sup>5</sup> In the first page of his contribution Kalecki pays homage to Aftalion defining him as the “leading exponent” of the type of theory to which also his own theory is declared to belong, “one which starts from investment processes and pays special attention to the time of construction of capital equipment” (ibid., p. 67). He immediately clarifies, however, that his theory is spoiled by “the false assumption that productive capacity remains constantly full employed” (ibid., p. 67). This criticism is further expanded at the end of his *Essay*.

from monetary dynamics in the long-run. The exponents of this approach noticed, at least since Hume, the recurrence of economic fluctuations explained in terms of temporary money illusion bringing about a certain degree of short-term rigidity of prices (one of the first and best accounts may be found in Hume (1742/1987)). Under these assumptions, an increase in the supply of money may stimulate the economic activity in the short run so long as money illusion persists. The growth of the real economy was instead explained in real terms by focusing mainly on the growth of population and technical progress.

Wicksell was the first professional economist to connect growth with business cycles explaining the latter substantially in real terms.<sup>6</sup> Therefore, since our prototype model explains business cycles and growth mainly in real terms, it seems natural to start by Wicksell the story of its genesis and evolution within the economic profession.<sup>7</sup> Wicksell analysis combined a general equilibrium perspective and an Austrian theory of capital (Böhm-Bawerk 1884) to show that monetary equilibrium in a developed financial economy depends not so much on the quantity of money and appropriate monetary policies as the quantitative theory maintained but also more and more on the coordination of choices on the inter-temporal allocation of real resources. Equilibrium between investment and saving decisions is guaranteed only when the monetary interest rate is equal to the ‘natural’ real interest rate, i.e. the “marginal product of capital” (Wicksell 1907, p. 214). He emphasized that such an equilibrium may be easily disturbed by changes in the natural rate reflecting technical change triggering a ‘cumulative process’. He stressed thus a crucial market failure as the market does not guarantee equilibrium between saving and investment decisions nor a rapid return to equilibrium when the latter is disturbed. This was perceived as a puzzle by mainstream economists that were stimulated to clarify its causes and implications. The impact of the cumulative process on the birth of modern macroeconomics cannot be overemphasized. This is true also of macrodynamics, the specific object of our essay. Wicksell developed

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<sup>6</sup> We have seen that a real approach to growth and cycles had already been taken by Marx but his powerful insights had been almost completely ignored by professional economists. The much more influential exponents of the “real bills doctrine” (such as the young Torrens, Took, Fullarton, James Mill, the young Stuart Mill) reversed the causal relation between money and real economy drawing implications on the explanation of inflation and on the most appropriate monetary policy but did not imply a theory of the dynamics of the real economy.

<sup>7</sup> For some interesting points on this matter, see the contributions collected in Jonung (1991), in particular the ones by Baumol (pp. 185-198), Brems (pp. 231-244) and Andvig (pp. 411-431).

the cumulative process not to explain the cycle or growth but rather the secular movement of prices in an economy increasingly dominated by the credit behaviour of commercial banks (Laidler 1999, p. 28). However his insights had a great impact both on the methodology and contents of macrodynamics as we will see in what follows.

Two main streams of thought sprang directly from the Wicksell muddle: the Austrian school of macroeconomics (Mises, Hayek) and the Stockholm school (Lindahl, Myrdal, Ohlin and Lundberg). We will briefly consider them exclusively from the point of view of the genesis of the prototype model.

#### **4.2 The Austrian School**

The Austrian school reacted to the challenge of the cumulative process by re-establishing the role of market self-regulation also in the coordination of inter-temporal choices. This led to the sketch of a macroeconomic theory very influential in the early 1930s with characteristics opposite to those of emerging Keynesian macroeconomics.

Mises argued that, taking account of all the effects of a change in the natural rate of interest, the self-regulating virtues of a competitive market could be re-established. He admitted delays in the re-adjustment process due to inappropriate credit behaviour of commercial banks leading not to a cumulative process but to “a theory of business cycles” (Mises 1924/1934). Hayek soon developed the Mises perspective showing that the credit provided by banks to the entrepreneurs (under the hypothesis of full employment equilibrium) brings about forced saving on the part of consumers undermining the co-ordination of inter-temporal allocation. The policy implications were strictly non-interventionist, in the conviction that the expansion of money in circulation or credit or public works would address the symptoms and not the causes prolonging the depression delaying the inevitable real adjustment. The causes are the distortions in the intertemporal structure of production whose fixing requires time and may be realized only in a decentralized way by entrepreneurs.

Laidler (1999, p. 49) has already shown the polar opposition of the Austrian school to the IS-LM model. We may show its polar opposition also to the multiplier-accelerator models. As for the accelerator, Laidler (1999, p. 42) remarks that even a young colleague and follower of Hayek at LSE criticized him because the investment expenditure does not increase with the increase of consumption as “the two groups of industries are *competing* for the services of the ultimate factors of production” (Durbin

1933, p. 137). Although Durbin was a sympathetic reader he had to conclude that Hayek analysis was appropriate only under full employment (Durbin 1933, pp. 139-141). The resilience of the price system and of the rate of interest itself to a disequilibrium induced by credit does not leave room for a readjustment on the side of quantities such as that described by the multiplier. In addition the crucial role of the de facto assumption of full employment in his argument is inconsistent with the existence of the multiplier.

### **4.3. The Stockholm school**

The *Stockholm school* addressed the same muddle addressed by the Austrian school taking, however, a completely different direction progressively converging towards the multiplier-accelerator model. While the Austrian school tried to played down and tame the disequilibrium process implied by the cumulative process effacing the contradiction with their theoretical convictions (general equilibrium), substantive convictions (self-regulating nature of a competitive market) and policy propensities (*laissez faire*), the Stockholm school took this disequilibrium process very seriously and tried to analyze it by exploring a more innovative research programme capable to study its details and implications. According to Wicksell, as synthesised by Myrdal (1931/1939, pp. 37-38), the ‘normal rate of interest’ that guarantees monetary equilibrium must

“( 1 ) equal the marginal technical productivity of real capital (i.e. the ‘real’ or ‘natural’ rate of interest); ( 2 ) equate the supply of and the demand for savings; and, finally, ( 3 ) guarantee a stable price level, primarily of consumption goods”

Wicksell assumed that these three criteria are equivalent but he did not succeed to prove it. The Austrian school focused on the first criterion in the conviction that coming back to the original fully worked out Böhm-Bawerk concept of capital and developing it further the Wicksell shortcomings could be overcome. They struggled thus to show that the first condition properly understood implies the second while Hayek maintained that complying with criterions 1 and 2 implies stable prices in a stationary process or diminishing prices in case of growth. The Stockholm school, on the contrary, discarded the first condition in the conviction that it is not possible to give a rigorous definition of the marginal product of capital for the entire economy independently of the market rate of interest. They focused thus on the second condition properly redefined on the basis of a Fisherian theory of capital and this brought at the forefront the crucial role of

expectations; this implied open-ended implications from the point of view of the third criterion and the policy implications of the analysis.

As mentioned above the crucial move of the Stockholm school was the rejection of Böhm-Bawerk theory of capital in the conviction that it is impossible to provide an aggregate measure of the quantity of capital (the average period of production) and of the marginal productivity of capital (natural rate of interest) independently of the market rate of interest. The way out was found, following Fisher, by assuming that the nominal value of the capital stock depends on the expected future streams of nominal income discounted at the nominal rate of interest. The expectations assume thus a crucial role in macrodynamics. The fact that they took seriously at the same time the role of disequilibrium and expectations raised awkward methodological problems. In disequilibrium, generally speaking, expectations cannot be assumed to be correct. There is thus a crucial problem of revision of expectations in order to eliminate the systematic errors. They were the first to explore in a rigorous way the two basic methods to treat this awkward problem. Lindahl introduced the method of temporary equilibrium that describes a macrodynamic process as a sequence of equilibrium states “in the sense that there will be equality of demand and supply during the period” (Lindahl, 1930, tr. 1939, p.159), while the factors of change are kept exogenous and are allowed to act only between the periods. Lundberg immediately observed that this method spoils time of any essential role in economic analysis: only if individuals are mistaken in their anticipations [would] the time distances ... influence price formation (Lundberg 1930/1995, p.34, as quoted by Laidler 1999, p. 60). Lindahl himself admitted that this approach clouds a ‘latent disequilibrium’ from the long period point of view and limits the analysis of dynamic sequences to factors taken as exogenous, in particular the expectations themselves (Lindahl 1939, p. 69, quoted in Laidler 1999, p. 61).<sup>8</sup> For this reason the exponents of the Stockholm school devised an alternative method meant to analyse directly disequilibrium macrodynamics. In this method the dynamic process is propelled basically by an endogenous factor, i.e. the systematic mistakes revealed by a gap between ex ante desired values of the crucial variables and their realized values ex

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<sup>8</sup> The temporary equilibrium approach was resumed and somewhat extended by Hicks (1939), and subsequently by the so-called disequilibrium macroeconomics (Clower 1965; Leijohnufvud 1968; Silvestre 1991). This method may be also considered as the ancestor of the equilibrium approach of New classical macroeconomics that however completely eliminates the distinction between short and long period.

post. This method was first suggested by Lundberg (1930/1995), further advanced by Myrdal (1931/1939) and fully developed by Lundberg himself (1937). Also this disequilibrium method had its shortcomings, as promptly recognized by the exponents of the Stockholm school themselves. In particular the choice of the unit period risked to be arbitrary and relative to the choice of the problem. The first dynamic formalization of the multiplier and of the interaction between multiplier and accelerator adopted this method reaching a remarkable degree of sophistication in Lundberg (1937). For this reason some interpreters consider Lundberg as the real originator of the multiplier-accelerator model.

#### **4.4. Was Lundberg (1937) the originator of the multiplier-accelerator model?**

Many authors (see, for example, Berg 1991; Brems 1991; Siven 1985, 2003) have stressed that the roots of the MA model are to be found in the work of members of the Stockholm School, in particular Ohlin – who in 1934 described the interaction between the multiplier and the accelerator principle but only verbally<sup>9</sup> – and Lundberg (1937) – who formalized it, in five different settings, in terms of difference equations. Lundberg however did not find an analytical solution of his models. Consistently with his method of sequence analysis, he relied on particular sequences generated by the models using illustrative values of the parameters. His main purpose was to describe the expansion phase of the business cycle and to show that there are endogenous forces that will sooner or later interrupt it. First of all, in a footnote at page 185, there is a short mathematical exposition of proportional growth given constant output-output ratio and a constant average propensity to save (meant to illustrate Cassel's verbal discussion) which is in essence identical to the Harrod-Domar model of growth. Indicating total income with  $E$ , savings with  $S$ , the average propensity to save with  $s$ , investment with  $I$  total capital with  $K$  and the fixed relation between capital and income with  $v$ , Lundberg find the condition of dynamic equilibrium between savings and investment by noticing that:

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<sup>9</sup> According to Siven (2003, p. 323), Ohlin used the multiplier to determine national income starting from Kahn (1931) and the accelerator to determine investment probably starting from Clark (1917). In doing this he was probably inspired by the debate between Frisch and Clark in the *Journal of Political Economy* as reported in Frisch (1933).

$$S(t) = sE(t) \qquad I(t) = S(t)$$

$$K(t) = K(0) + \int_0^t I(t) dt \qquad K(t) = vE(t)$$

so that:

$$vE(t) = K(0) + s \int_0^t E(t) dt$$

or:

$$vE'(t) = sE(t)$$

from which we find:

$$E(t) = Ce^{\frac{s}{v}t}$$

But, what happens if we were somewhat outside the growth trajectory? This is discussed by Lundberg in the following two sections, first with reference to investment in working capital, then to investment in fixed capital. In both cases, the result is a dynamic multiplier-accelerator model giving rise to a second-order difference equation (see Lundberg 1937, p. 201 and 209 respectively). The Keynesian multiplier is clearly there.<sup>10</sup> Moreover, for example in the second cases considered by Lundberg, i.e., the case of expansion of the production of consumption goods determined by investments in working capital, the investment in working capital  $I$  is formalized as:

$$I_t = kC_t - K_{t-1} \tag{2}$$

where  $C$  is total business expenditure on the production of consumption goods and  $K$  the stock of working capital. Taking account also of the other relations of the model, (23) can be written as:

$$I_t = (1+k)(E_{t-1} - E_{t-2}) \tag{24}$$

which makes it explicit that investment in working capital is determined by an accelerator mechanism.<sup>11</sup>

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<sup>10</sup> After having studied the effects of an autonomous increase in consumption, Lundberg (1937, p. 195) concludes by saying: "This formula evidently expresses the same relation as Keynes' theory of the multiplier". We should keep in mind that the book, as it is testified by the Foreword, was finished in December 1936 so that Lundberg had had the time to read the *General Theory*.

<sup>11</sup> (23), on the other hand, is in the form of the so-called 'flexible accelerator'. Cfr. Goodwin (1948).

The final dynamical equation of the model turns out to be a second order difference equation for the derivation of which it is crucial the assumption of the so-called ‘Lundbergian lag’, reflecting the lag it takes to adjust output to a change in sales.

Thus, Lundberg’s model gives us the first example of a formalization of the multiplier-accelerator interaction.<sup>12</sup> So, the important question arises: why is it so seldom quoted in the literature on business cycle modelling? Or: why did it not get the recognition it deserved? This is a much debated topic (see, for example, Siven 1985). Our impression is that above all this happened for twofold motive. First, as we have already stressed, Lundberg never tried to find the general solution of the systems of equations and relied on specific examples. Samuelson’s analysis, on the contrary, succeeded in supplying a full characterization of the dynamics of his model.<sup>13</sup> Second, Lundberg never recognized the possibility of disinvestment in the downward swing and assumed that the above equation (25) holds only when  $E_{t-1} > E_{t-2}$ , otherwise it simply becomes  $I_t = 0$ . Thus, his model is not a complete one (Berg 1991, p. 206).

## 5. The Marshallian tradition and Harrod’s crucial contribution

### 5.1 *The Marshall connection*

The existence of a Marshallian tradition in macrodynamics has been recognized since long. The Marshallian tradition dominated the contributions of the British economists up to the 1930s and extended its influence much beyond the British borders as Ohlin witnesses for the Stockholm school (in 1932: see the quotation in Laidler 1999, p. 75) or we will see in the American tradition. It is however difficult to characterize the distinctive features of this tradition since the British contributions to macrodynamics in

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<sup>12</sup> This means that to some extent Lundberg preceded Samuelson (1939a,b). In addition, as stressed by Siven (2003), it might even inspired it! We know that Lundberg, thanks to a Rockefeller Foundation fellowship, spent two years in US in 1931-1933 and met Alvin Hansen who, later, got a copy of his dissertation. Hansen might have got from there the idea that led him to work out the numerical exercise that inspired Samuelson. Some indication in this direction is the fact that Samuelson (1939a, p. 78, f. 1) writes: “It may be mentioned in passing that the formal structure of our problem is identical with the model sequences of Lundberg.”

<sup>13</sup> We should mention, however, that Samuelson’s formal approach is fully effective in providing analytical solutions only when the model takes the most elementary (*linear*) form. Thus, as stressed by Baumol (1991, p. 185), “the ostensible superiority of the formal approaches in terms of the greater generality of their results is to a considerable degree an illusion.”

the above period are extremely rich and variegated. We may say however that the applied work of Marshall on economy-wide problems were characterized by a few basic features that proved to be very influential on his successors. First, he took into account the psychological determinants of human behaviour. This led him in particular to reformulate the equation of exchanges in terms of stock demand of money, an approach that opened the way to a systematic analysis of the role of the psychological factors of agents' behaviour. Second, he always emphasised the importance of the institutional factors. This led him in particular to recognize the growing role of commercial banks in determining the transmission of a monetary impulse on the economy. Third, differently than in his theoretical contributions much more long-period oriented, he gave a role to aggregate demand in the short period as a possible limiting factor of economic activity (everybody knows the scissor metaphor). Fourth, he described business cycles as disequilibrium processes characterized by price stickiness; he underlined in particular the slow adjustment of the market rate to the equilibrium rate of interest exerting a recognized influence on Wicksell, and of real wages to their equilibrium level capable to avoid unemployment. Finally, consistently with the above points, his policy advices never conformed to a crude *laissez faire*. The acknowledgment of the existence of market failures, at least in the short period, led him to examine the possibility of state interventions to counteract market failures.

These points were developed by different authors well before the 1930s. The role of psychological factors was analysed in particular by Pigou. The demand for money was thoroughly developed by Lavington who anticipated Keynes theory of liquidity preference. Specific institutional aspects were analysed by different authors. In particular Hawtrey analysed the evolving role of credit and its intrinsic instability. The role of aggregate demand and of the income-expenditure interaction was especially recognized by Robertson. Pigou developed for the first time a microeconomic theory of market failures explained in terms of externalities. Most exponents of the Marshallian tradition supported in this period an attitude of active policy interventions to avoid or counteract policy failures: active monetary policy to accelerate the convergence of the market rate of interest to its equilibrium value, interventions to reduce price stickiness, public works to counteract depressions

The prevailing approach of the Marshallian school was favourable to the systematic introduction of the multiplier and of the accelerator in macrodynamic analysis but this occurred in a convincing and influential way only in the 1930s.

### **5.2. The Multiplier: Kahn, Warming and Keynes**

In the case of the multiplier the first analytical treatment that made an impact on mainstream economists is due to Kahn (1931). Its precise formulation was already implicit in the *Tableau économique* by Quesnay and, as we have seen, in the reproduction schema of Marx. It was then made explicit at the turn of the 19<sup>th</sup> century by a few non-professional economists (Dimand 1988), and in particular by the German-American businessman and economist Nicholas Johanssen who used the term ‘multiplying principle’ in his fairly precise discussion of the effect of investment expenditure on the economic activity (Johanssen 1908).<sup>14</sup> However, no one questions that the modern version of the multiplier that became a crucial component of Keynesian macroeconomics was introduced by Kahn. He wanted to show that in a situation of unemployment an increase in public expenditure would employ otherwise unemployed workers and that would start a sequence of increases in all the sectors of the economy as the additional income of the newly employed workers would translate in higher expenditure and so on. He clarified that “the ratio of secondary employment to primary employment” could be conceived as the finite sum of a geometric progression  $k / (1 - k)$  where  $k$  represents the percentage of the new generated income spent in consumption goods. He clarified that the precise formula of the multiplier depends on the fact that  $k$  may be safely assumed to be less than one. He also emphasized that this simple formula that ignores the effects on prices is fit “at times of intense depression” while “at normal times, when productive resources are fully employed ... building of roads carries with it little secondary employment and causes a large rise in prices” (Kahn, 1931, p. 182). Kahn applied his employment multiplier to an open economy and emphasized the import of goods as main leakage. In 1932 Jens Warming, a Danish statistician, clarified that Kahn’s multiplier works as well in a closed economy provided that we understand that the main leakage that assures a relatively quick convergence to

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<sup>14</sup> It is interesting to recall that Keynes mentioned Johanssen in the *Treatise* (1930).

the finite-sum expression is given by the fact that the consumption propensity is typically quite inferior to unity.

Warming's insights were promptly received by Keynes and adopted in the version used in his work with the Committee of Economists of the Economic Advisory Council to the British Government (Keynes 1933). This version induced Keynes to complete a crucial step forward. Although Keynes had nothing to add to the Kahn-Warming multiplier from the analytical point of view he was the first to use it as the crucial determinant of income in the short run. As Goodwin (1947, p. 482) observed "he gave it the role it plays today, by transforming it from an instrument for the analysis of road building into one for the analysis of income building". This may seem rather ungenerous vis-à-vis Kahn, Warming and other predecessors, but emphasizes correctly the further step accomplished by Keynes in the use of the multiplier from the point of view of the underlying vision. To this end however he took the multiplier not as a process in time but as a timeless causal relation. Keynes 'telescoped' the process within a single timeless short period equilibrium. This gave simplicity and strength to his argument but severed the multiplier from the analysis of business cycles. For the same reason he excluded any possible analysis of the accelerator that requires a consideration of changes in the capital stock. Such a change was excluded by the definition of short period (Keynes 1936). Many economists sympathetic with the Keynesian revolution immediately felt that the new framework of macroeconomic analysis had to be extended from the short period to macrodynamics. The first and most important step was moved immediately by Harrod (1936) who reasoning in terms of dynamic processes understood the importance of modelling the multiplier as a dynamical process in time coupling its effects with those of the accelerator (that he called the 'Relation').

### ***5.3 The interaction between the multiplier and the accelerator: Harrod's crucial contribution***

There is a wide agreement on the fact that Harrod was the first British economist who analysed the interaction between the multiplier and the accelerator in a fairly rigorous way (Harrod 1936, 1939). There is also a wide agreement, however, that Harrod's contributions did not succeed to elaborate a fully rigorous analytical model of such interaction. In particular it is emphasized that he did not understand that for particular values of the parameters the MA interaction is sufficient to explain self-

sustaining cycles (see, e.g., Hertje and Heemeijer 2002 and Besomi 2001, 2003). In this section we intend to submit these opinions under critical scrutiny, keeping in mind the distinction between vision and analysis as we have specified it in the introduction. At the same time we have to take account of Harrod's distinction between theory, models, and application of the theory. Differently from the point of view that was emerging in the 1930s and was going to conquer mainstream economics within a few decades, the theory is not identified with, or flattened against, analytical models. Harrod (1939).

The accelerator was present since long in the British tradition in the general and vague sense of a complementarity between demand of consumption goods and demand of investment goods (as observed by Durbin, retro), but did not play a role as more or less formalised piece of analysis as it did in the American tradition (see section 6). The only exception was Hawtrey who analysed the accelerator as early as in the 1913. For this reason he has been often considered as one of the main originators of the accelerator together with Aftalion (1913) and Clark (1917).<sup>15</sup> However, he did not come back on the issue in his following influential contributions on monetary business cycles so that this specific contribution failed to attract the attention of his colleagues. As a matter of fact the accelerator had been introduced much before by two British researchers Mummery and Hobson as early as in 1889. They provided a fully worked out numerical illustration of the accelerator relationship (Mummery & Hobson 1889, pp. 85-86). This contribution however was ignored by professional economists because of their unconventional point of view that eventually produced the expulsion of Hobson from the academia. We may thus say that Harrod was the first in the British academic tradition that took seriously the accelerator as essential piece of analysis for a satisfactory explanation of business cycles. This is confirmed by the terminology adopted by Harrod. He felt so little constrained by a previous tradition that he called the accelerator with the idiosyncratic name of the 'Relation' (Harrod, 1936).<sup>16</sup>

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<sup>15</sup> Another significant exception was Bickerdike (1914), an outsider who managed to publish different articles on the *Economic Journal* including a fundamental one that clearly defined in algebraic terms the accelerator. His contribution, however, went completely unnoticed within the professional economists.

<sup>16</sup> The reason for this change of terminology is explained by Harrod himself (1939, p. 14, n. 1): "a steady state of increase of demand, which is our first matter for consideration in dynamic theory, and a major effect of which is represented by the 'Relation' should be regarded as a velocity. Acceleration would be a rate of change in this". However, since *An Essay in Dynamic Theory*, Harrod 'reluctantly' adopts the consolidated terminology because "the use of the expression Acceleration Principle in the sense of my relation is rapidly accelerating in current literature" (ibid.).

As for the multiplier he was an early contributor to its development. In his *International Economics* he developed in a sophisticated formalised way the multiplier of foreign exchanges anticipating the textbook presentation of the Keynesian multiplier with fully exogenous investment (see Pugno 1992, p. 147). In the same work he announced his research programme on macrodynamics that he was going to pursue steadily throughout his entire career. He maintained that the field of macrodynamics had still to be developed since the early promising advances of classical economists had been interrupted by the neoclassical economists. He claimed that the revival of macrodynamics had to be founded on the interaction of the multiplier and the accelerator principle (Harrod 1933, p.158 and p.164). We should thus give the deserved credit to Harrod not only as an important early contributor to the multiplier but as the first scholar who understood the potential of the MA interaction for macrodynamics. Harrod's priority is arguable not only within the British tradition but also more in general. The contemporaneous debate between Frisch and Clark in *Econometrica* ascertained that the multiplier alone could not provide an explanation of business cycles and that another interacting mechanism was needed. It is surprising that no one suggested the accelerator as a serious candidate (Frisch 1931, 1932a,b; Clark 1931, 1932). As for the Swedish tradition, the first explicit intuition of the potential of MA interaction emerges in Ohlin in 1934 but he may have drawn it from Harrod.

This research programme is confirmed by Harrod in the following year in his work on imperfect competition (1934) and starts to be implemented in his *Trade Cycles* (1936). This work is underestimated and is rarely mentioned in the histories of business cycle theories. This depends on the fact that the argument is not formalised and is sometimes obscure. However in this work the dynamic implications of the MA interaction start to be worked out leading to a few remarkable intuitions. This was recognized later on by some of the main exponents of macroeconomic dynamics such as Samuelson (1939b), Kaldor (1940, p. 190n), Hicks (1950, pp. 6-10) and Goodwin (1951, p. 142n). In this contribution Harrod's research programme on macroeconomic dynamics was further clarified. He conceived of macrodynamics as one of the main sections of economic theory that has to supplement neoclassical microeconomics (extended to the case of imperfect competition) and Keynesian macrostatics. The first step of this research programme, taken already in 1934, is the idea that the business

cycles are oscillations not around the static equilibrium of traditional theory but around a trend of ‘uniform’ and ‘steady’ advance. In his opinion, only in this way we may provide a satisfactory explanation of the fluctuations that characterize the modern capitalist economies. At the beginning he conceived of this equilibrium as a dynamic version of traditional equilibrium (Harrod 1933, 1934). He realised then that the steady advance should not to be confused with the actual path of the economy that may crawl along it only during the boom (Harrod 1936). The intense exchange with Keynes before and after the publication of the *General Theory* and of his own *Trade Cycles* in the period 1935-1938 led him to part radically from traditional equilibrium theory (see on this point Pugno 1992), suggesting a conceptual framework that would have guided his research on dynamics for the rest of his life (Harrod 1939). First of all he distinguished between two equilibrium paths: an endogenous path, christened ‘warranted’ rate of growth and an exogenous path that he calls ‘natural’ rate of growth. The warranted rate of growth assures the equilibrium between aggregate demand and supply taking into account the multiplier and the accelerator principle so that “there will be a unique value of  $G$ , the growth rate of the economy, which is consistent with people saving that they want to save and having the capital goods that they require for their purposes” (Harrod 1973, p. 19). The warranted rate of growth, generally speaking, is not characterized by full employment that characterizes instead the ‘natural’ rate of growth that “is the maximum rate of growth allowed by the increase of population, accumulation of capital technological improvement and the work leisure preference schedule, supposing that there is always full employment in some sense.”. The effective rate of growth of a certain economy is sharply distinguished from both equilibrium concepts. The dynamic theory of Harrod is thus a disequilibrium theory where the equilibrium paths play only the role of necessary references for the analysis of the effective rate of growth. In addition Harrod shows that the warranted path is dynamically unstable, although this instability is bounded by a few crucial nonlinearities.

The *Essay* is a very original ‘framework of concepts’ that teaches how to ‘think dynamically’ keeping together business cycles and growth. In addition, some of the crucial new concepts, in particular the warranted and the natural rates of growth and the ‘instability principle’, receive a preliminary formalization crucially based on the “marriage of the acceleration principle and the multiplier theory” (Harrod 1939, p.16).

This time, the Harrod's outline of a 'dynamic' theory did not fail to have a profound impact on his academic colleagues, although it was not what he expected to have. The *Essay* was taken to be the first important step to build a Keynesian growth theory understood as set of analytical models set to explain, compare, forecast and control growth trends in developed economies disregarding any sort of fluctuations around them. This view betrays Harrod's intentions on at least two basic points: it flattens theory on analytic models based on dynamic equations and, also in consequence of this, it severs growth from cycle. We know that a model based on the interaction between the accelerator principle and the multiplier may account for both growth and business cycles, confirming to some extent the deep intuition of Harrod, but only within different specifications of the model, at least as far as leads and lags between endogenous variables is concerned, since these hypotheses change the order of the dynamic equation. Harrod disliked an excessive emphasis on the assumptions about leads and lags that he perceived as largely arbitrary, if not ad hoc, assumptions, what we could today call 'free parameters'. In his advocated theory of dynamics these assumptions had to be managed in different contexts by a sound conceptual framework. He thus never accepted the bifurcation operated in the late 1930s within Keynesian macrodynamics between business cycle and growth branches. Quite paradoxically, he exerted an influence on growth theory as one of the authors of the Harrod-Domar model while he was almost completely disregarded as contributor to business cycles theory.

Notwithstanding this heavily reductionist interpretation of Harrod's approach to dynamics, there was still a disturbing feature that determined a further shift away from his vision of macrodynamics in the following years: the instability principle. He was able to show, without using a mathematical demonstration but through substantially correct qualitative arguments, that the warranted rate of growth "represents a moving equilibrium, but a highly unstable one" (ibid., p. 22). He commented emphatically: "Of interest this for trade-cycle analysis!" And again he was right because he introduced nonlinearities that translate this instability in cyclical fluctuations, although not very regular. The instability principle inspired in the two following decades a rich literature meant to eliminate or weaken the dynamic instability of the model. But in the end it brought to the demise of the MA models. A crucial step in this direction was the model of Solow (1956) that introduced the 'neoclassical model of growth' although it is

presented as a generalization of the MA model obtained by assuming substitution between capital and labour. The underlying motivation for stabilising the steady-state growth is clear. As Solow himself openly maintained, in the real world the divergences from steady state growth are small, stochastic and hardly explosive (see Pugno 1992, p. 183). Although he did not deny that these cyclical deviations could be interpreted in terms of disequilibrium, this prepared the subsequent step that interpreted the deviations from equilibrium as exogenous and stochastic. This view led to the ‘equilibrium business cycle’ suggested by Lucas in the early 1970s (Lucas,1981) and the ‘new growth theory’ founded by Romer (1986) and Lucas (1988).

Summing up we believe that Harrod understood the potential of the MA interaction in explaining disequilibrium and instability of an advanced capitalist economy better than any other academic economist, and that his insights are still valuable to understand the intrinsic instability of fluctuating growth in developed capitalist countries.

## **6. The American tradition and the birth of the Samuelson’s prototype model of business cycle**

### **6.1 *The American tradition***

The prototype model of business cycles springs from the American academia as the outcome of the personal link between a senior and recognized scholar of business cycles, Alvin Hansen, and a very promising young pupil, Samuelson. This happened in Harvard not by chance. In United States there was a rich and variegated tradition of business cycles studies open to foreign influence, in particular the Cambridge tradition, but with a high degree of originality. This was not a tradition in the sense that the different research streams had a common founding father as in the case of the Marshallian and Wicksellian traditions; however we can identify a few distinctive characters. We mention two that are of particular interest for our purposes. The first one is a particularly strong institutionalist tradition that had been introduced and developed by Veblen at the turn of the century and that continued to exert a sizeable influence on business cycle research, in particular through the works of Mitchell (1927). Also scholars not classifiable as institutionalist show here and there the influence of this

stream of thought. This is true in particular for Hansen who “was very much a product of the same American institutionalist tradition in cycle theory whose leading exponent was Mitchell” (Laidler 1999, p. 203, f. 22). In particular, Hansen draws from Mitchell (1923) a clear understanding of the crucial role of the accelerator in explaining business cycles. Hansen stressed mainly its technical underpinnings emphasising the relationship between the flow of current production and the required stock of capital goods. However, the monetary approach to macrodynamics based on the equation of exchanges clouds this as well as other crucial real conditions of macrodynamics.

The second peculiarity was a confidence on the self-equilibrating virtues of the market much deeper than in the Cambridge and Stockholm traditions with the only exception of the Austrian school. This confidence is particularly evident in the work of Fisher, perhaps the most eminent American economist of the first part of the century. He believed in the self-regulating character of the market and maintained that business cycles were just a statistical artefact. This led him to rehabilitate the quantity theory of money in an updated version that takes account of the evolution of the monetary system. His optimism in the virtues of the market misled him to the point that in September 1929 when the real economy was already receding he expressed the view that the stock-market boom was going to last, while in 1930 he insisted that the market collapse was just temporary. He repeated in 1933 what he had argued before the crisis (in particular in 1923) that “the old and apparently still persistent notion of ‘the’ business cycle as a single, simple, self-generating cycle ... realized historically in regularly recurring crises, is a myth” (Fisher 1933, p. 338). Although few other economists followed his lead on this point, most of them, including Hansen, believed that “the effect of any initial disturbance would soon wear off after a very few oscillations of rapidly diminishing amplitude” (Hansen 1927, pp. 202-203). The persistence of economic fluctuations may thus be explained only by “new disturbing factors” (ibid., p. 198). Fisher recognized however that “it is absurd to assume that ... the variables of the economic organization ... will ‘stay put’ in perfect equilibrium, as to assume that the Atlantic Ocean can ever be without a wave” (Fisher 1933, p. 339). He had also to recognize with hindsight that the boat of the economy may capsize under the influence of these waves. In consequence of the debt-deflation cumulative process “we have the great paradox which, I submit, is the chief secret of most, if not all, great depressions: *The more the*

*debtors pay, the more they owe*. The more the economic boat tips, the more it tends to tip. It is not tending to right itself, but is capsizing” (Fisher 1933, p. 344, italics in original). He believed however that this problem could be solved by imposing a 100% reserve requirement against the monetary liabilities of commercial banks. The optimism of Fisher was shared, although in more moderate form, by most other American economists in the 1920s and early 1930s. This may be explained by the fact that the first world war had been shorter and its consequences much less dramatic than in Europe. In addition the ‘roaring 1920’ were years of prosperity and full employment quite different by the 1920s in Europe plagued by hyperinflation and secular stagnation. Finally the American economy was becoming the leading economic power by relying on *laissez faire*.

An exception to the conventional American optimism was the underconsumptionist school of thought. Its main exponents, Foster and Catchings, were not academic economists and suffered less from mainstream habits of thought. This led them to anticipate, building on ideas of Veblen and Hobson, some basic ideas of the Keynesian revolution. They adopted an income-expenditure representation of the circular flow of income showing that “a penny saved is sometimes a penny lost” (Foster & Catchings 1925, p. 400). Their argument on the chronic lack of aggregate consumption to assure full employment equilibrium is based on a rudimentary multiplier mechanism probably borrowed by Johanssen (1908; see Laidler 1999, p. 173). The policy implications of this approach led its exponents to justify a systematic intervention of the state mainly through public-works expenditure. Their ideas had influence also on scholars in the Academia. The most influential academic underconsumptionist was no doubt Paul Douglas, professor at the University of Chicago who stressed in particular that “each dollar spent on public works would ... have a multiplicative effect in stimulating business” (Douglas 1933, p.12).

We may conclude that when the Keynesian revolution arrived in America found a fertile terrain for its success. The persistence of the economic contraction had much moderated the optimism of American economists strengthening the influence of economists, such as the institutionalists and underconsumptionists, more critical of the market and more orientated towards the real determinants of macrodynamics but lacking a convincing theoretical framework. This is what Keynes managed to offer: an

intriguing, simple, but relatively sophisticated approach from an insider at the same time of the Academia, finance and public service communities. The simplicity of the basic argument was suitable for a mathematical formalization to draw precise policy indications and seemed ideal for econometric testing. However to get all its potentialities from the Keynesian approach his heuristic static model sketched in the GT had to be developed in terms of rigorous dynamic models

## **6.2 The relationship between Hansen and the young Samuelson**

Samuelson (1939a,b) is often taken to be the first rigorous mathematical formulation of the interaction between the multiplier and the accelerator principle. As such, the Samuelson model is a landmark in the history of macrodynamics. We have seen it as the culmination of a genetic process. Macrodynamics would not be the same afterwards in both its main articulations, business cycle theory and growth theory. Its immediate genesis has been reconstructed by Samuelson himself in 1959 and, as we have already stressed in footnote 2, has recently been again at the centre of the attention in the debate on the pages of *History of Political Economy*. The ‘outbreak factor’ of the debate was the discussion by Heertje and Heemeijer of Samuelson’s attribution of the original accelerator-multiplier model to Alvin H. Hansen, his mentor at Harvard, rather than to Harrod. The fact is that in Samuelson (1939a), Harrod is not even mentioned whereas in Samuelson (1939b) we find some discussion of the literature, including Harrod (1936), who however is not celebrated as the main inspirer. This appears rather odd at first sight although a possible explanation is to be found in Hansen’s attitude towards Harrod’s work, not much positive as testified by his review of Harrod’s book on the *Trade Cycle* (Hansen 1937 and 1938, Ch. 2). As consequence of this, we can image that the name of Harrod was ‘unmentionable’ by a young researcher working under Hansen’s shield at Harvard. Harrod is however behind the scene in Samuelson’s contributions as is testified by the fact that, even in Samuelson (1939a, p. 75; original emphasis), for example in the sentence:

“Professor Hansen has developed a new model sequence which ingeniously combines the multiplier analysis with that of the *acceleration* principle or *relation*”

also Harrod’s own terminology is used (cfr. Besomi 2003, p. 315n).

The best way to understand the reason for Samuelson's position is to describe briefly how Hansen assimilated the Keynesian position. Although Heertje & Heemeijer (2002, p. 211) describes Hansen in 1939 as an "established Keynesian economist", it turns out that in that year he had just finished to assimilate the Keynesian position through a long process of *intellectual transformation* (see Barber 1987).

When the *General Theory* was published, Hansen was already a prominent professor of economics whose pattern of thought "had already been cast in a different mold" (Barber 1987, p. 191).<sup>17</sup> In fact, he was one of the few American economists of prominence of his generation that made the shift to Keynesianism and therefore it is important to understand how this happened. Hansen met Keynes for the first time in 1934 in USA, but he was aware of his work already some years before as it is testified by the numerous reference to Keynes's work we find in his contributions since the late 1920s. However, his orthodox neoclassical view on fiscal policy was such that he found very unconvincing all apparatus of Keynes's multiplier. As stressed by Barber (1987, p. 199), and as testified by the very critical review of the *General Theory* that Hansen published in 1936, when the latter appeared "the distance between the perspectives of the two men was as wide as, if not wider than, it had ever been." However, the situation changed drastically between 1936 and 1938, year in which Hansen published a collection of his essays containing a revised, less critical, version of his review of the *General Theory*. Among the factors that can explain Hansen's shift of opinion, one very important was his move to Harvard where he started to collaborate with John Williamson in the graduate seminar on Fiscal Policy. In this seminar, Hansen started to work on a possible explanation of the recession of 1937. In the process of solving this problem, he arrived very close to the Keynesian account of income determination, although he was using the accelerator but was still sceptical of the multiplier. However, thanks to discussions with students and colleagues (included the young Samuelson) in the Fiscal policy seminar he arrived at the end at a full acceptance also of the multiplier. As summarised by Barber (1987, p. 205), we can then conclude that "[w]hen the nature of the problem changed, he could change as well. His effort to understand the recession of 1937-38 heightened his appreciation of the merits of Keynesian analytical tools".

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<sup>17</sup> In the 1920s, Hansen followed two lines of research, mainly concentrating on business cycle theory and labour economics, and "[t]he bulk of his early writing (...) fitted into an orthodox neoclassical framework" (ibid., p. 192).

Thus, his influence on Samuelson worked through the Fiscal policy seminar and, to a certain extent, the influence was reciprocal.

As recollected by Samuelson (1959), at this point Hansen's problem was mainly to analyse the 1937-38 recession by trying to 'dynamize' existing Keynesian models. As a result of this, he assumed (1) that, according to the multiplier, consumption was half of *yesterday* income and (2) that, on the basis of the acceleration principle, the level of capital was twice the level of consumption and worked out a numerical example to find out the resulting behaviour of national income in time. Doing this, he realised that national income initially was rising but that then at a certain point there was a turning point and income started to decline. *Was this a possible explanation of the 1937-38 recession?* It was only at this point that Samuelson entered the scene. In his own words (1959, p. 183; original emphasis):

“I took *Hansen's* model, recognized its identity to a second-order difference equation with constant coefficients, and proceed to analyze its algebraic structure.”

It was not too difficult for Samuelson to realise that, by chance, Hansen had taken values of the parameters such as to give rise to a cyclical solution of the difference equation with constant amplitude and thus that Hansen's example was “able to generate a succession of never-ending expansion and contractions”. Generalising the example, it was easy then for Samuelson to show all other possible cases and to find all possible combinations of the parameters such that the model could generate damped or explosive cycles and converging or diverging monotonic dynamics. In his reconstruction of the genesis of the model, Samuelson is also very clear about the meaning he attaches to his simple, 'but deep enough' formulation. The latter, in his opinion (Samuelson 1959, p. 184), can usefully “serve as a pedagogical introduction to dynamic economic models”.

## **7. Concluding remarks**

As we have seen, in the 1930s there was a rapid convergence of different schools of thought, the most important ones in macrodynamics, toward a model describing the interaction between the multiplier and the accelerator principle. The most influential

prototype of analytical model of the business cycle, believed to be consistent with the Keynesian revolution, was eventually published by Samuelson in 1939, while in the same year Harrod published a prototype conceptual framework for the study of economic dynamics that initiated the Keynesian stream of growth theory. In our opinion the convergence of macrodynamics towards the MA interaction analysis, as well as its remarkable success may be explained by two basic factors. First the emergence of quantitative economics in the 1930s favoured the emergence and the success of the first rigorous analytical model of business cycles expressed in terms of functional equations, as well as of a semi-analytic conceptual framework for the study of economic dynamics that was soon translated in analytic terms. In addition, the simplicity of the underlying relations favoured the application of the emerging econometric models. Second the persistence of structural unemployment in the 1930s strengthened also among mainstream economists an approach based on effective demand, income-expenditure interaction and market failures that had already emerged before, particularly in the writings of outsiders as a reaction to the Great Depression at the turn of the 19<sup>th</sup> century. This family of macrodynamic models are based on endogenous dynamics that depends on structural factors of the system but the latter are highly simplified. This is in tune with the need of the simple and measurable policy suggestions, produced by the Great Contraction, on how to control it and avoid its repetition, and then on how to finance the war and, after the war, on how to promote steady growth.

The result of this evolutionary process, as often happens, cannot be read exclusively in terms of progress. It produced benefits but also losses and we are now in the position of discussing both of them. The emergence of mathematical models of business cycles and growth was no doubt an important step forward, one that brought rigour and clarity to the argument. However Harrod was right in resisting the confusion between theory and models. Although he was not able to work out a model explaining at the same time business cycles and growth his explorations of the interaction between them have inspired much of the subsequent research on non-linear macrodynamics. In addition, the interpretation and use of models requires a critical understanding of their meaning and limitations that has to come from qualitative theory. The reduction of theory to models induced the readers of Harrod (1939) also to ignore his path-breaking, but mainly qualitative, introduction of non-linearities and strong uncertainty.

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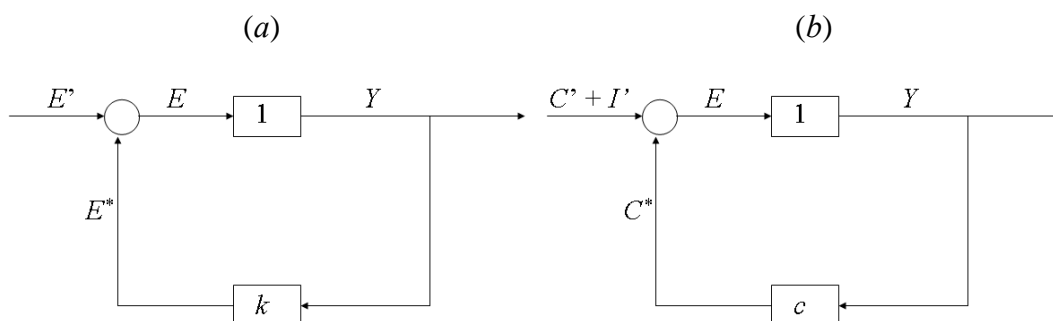
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**Fig. 1** (a) The basic multiplier and (b) Keynes's multiplier



**Fig. 2** The multiplier-accelerator interaction

