Going back to the origins of econophysics: Luigi Amoroso

[First draft]

by Gianfranco Tusset*

If nature has a purpose expressed by the principle of least action it is certainly not anything comparable with that of business man.

(Max Born, 1939)

> … as it is not possible to study the movement of each gas particle against the bottle containing it, but it is feasible to deal with pressure and gas mass by means of statistical averages, … so it is not possible to study the impulses of each individual, but it is workable to deal with the uniformities resulting from averages and sums of heterogeneous individuals' actions belonging to a given social circle or class, according to the casual distribution of such impulses (De Pietri-Tonelli 1931, 43, our translation).

Even if this statement is drawn from the 1931 *Course of Political Economy* of Alfonso De Pietri-Tonelli, its content does not appear far from claims of "econophysics," the recent research area that surfaced during the last decade of the twentieth century, as the following sentence shows:

> The founding idea, dating back to the works of Mandelbrot (1960), is that a trading market composed of a sufficiently large number of agents can be described using the laws of statistical mechanics, just like for a physical system composed of many interacting particles (Düring et al. 2008, 7).[1]

To materialize the fictitious nexus between the early-twentieth-century Italian economists/statisticians and late-century modern "econophysics," we remember that De Pietri-Tonelli was a Paretian scholar who theorized a market characterized by the heterogeneity of individuals,[2] which requests statistical methods, at that time common to both Italian statisticians[3] and economists. Interest in heterogeneity and aggregate phenomena may also be found in the best-known Paretian, Luigi Amoroso, a mathematician/economist devoted to dynamizing the general economic equilibrium. Finally, it must be stressed how present scholars engaged in wealth analysis,

* Department of Economics and Management, University of Padua. For correspondence gianfranco.tusset@unipd.it.

[1] An introductory book to econophysics is Mantegna and Stanley (2000).

[2] De Pietri-Tonelli did not raise doubts about his conception of heterogeneity: "Human society must be conceived in both space and time not as a cone-shaped entity, as scientific and statistical-economic analyses do, but as an heterogeneous complex of innumerable social circles, which are kept together by different features in a continuous process of making, changing, transforming …human society appears as a tangle … and heterogeneity, that is the inequality among individuals, characterizes both different circles and individuals belonging to the same circle." (1931, 5–6, o.t.)

[3] See (Prevost, 2002).

a subarea of econophysics, frequently quote both Vilfredo Pareto and Luigi Amoroso as the authors who first explained income distribution based on statistical data.[4]

Going back to the past, our concern is with heterogeneity, distribution, observational biases, aggregate variables, and thermodynamics insights, all aspects more or less recurring not only in the work of statisticians, such as Rodolfo Benini, Corrado Gini, Giorgio Mortara, and others, but also in the work of economists, first of all the Paretians, for example, De Pietri-Tonelli, Amoroso, and Giuseppe Palomba. By reviewing these authors, it cannot pass unnoticed that the then *quasi*-hidden features coming from statistics, biology, and mathematics were often grouped under a common heading, economic dynamics, the research field at that time under construction. Just the theories of economic dynamics will be the central thread of this paper, with particular attention to the work of Luigi Amoroso and to those among his theories presenting an empirical bias.[5]

The structure of the paper is as follows. Section 1 gives some insights into the Italian School of statistics appeared during the early decades of the twentieth century. Section 2 highlights the virtual links between Amoroso and econophysics. Section 3 briefly looks at Amoroso's supply theory. Section 4 describes the classical approach to economic dynamics. Section 5 presents the Amoroso theory of cycle, presenting it as an 'elastic' approach. Section 6 offers a view of his thermodynamics insights. Section 7 is devoted to some concluding statements.

1. Italian Statisticians and Economists

During the early twentieth century, in Italy among economists, interest in statistics was wide and was shared by economists closed to sociology and scholars of the Paretian general economic equilibrium. We mention De Pietri-Tonelli again, who, in 1921, wrote:

> Experimental researches about economic phenomena […] have not been numerous yet, but they are increasing according to the improvement of statistical analysis and methods. (1921, 9, o.t.)

This interest brought to work out numerous tools useful for quantitative analysis, some of them rather pioneering for that period. Really, as Schultz[6] pointed out, Rodolfo Benini was one of the first scholars to derive the demand elasticity of one or more goods from time series, making use of a method subsequently employed and developed by H. L. Moore. Moreover, as noted by Stigler,[7] Benini must be recognized as the first to use multiple regressions.[8] Other contributions to economic

---

[4] Other references to Maxwell's distribution of molecules can be found in 1934 Arrigo Bordin's textbook, *Lezioni di economia politica e corporativa:* "Mixed schemes [deterministic and probabilistic ones] are common into nuclear physics, molecular physics (Maxwell law) and in my viewpoint they should be employed with the aim of studying wealth and income distribution, that is, phenomena that do not answer to the hedonistic principle". See Bordin 1934, 51

[5] See Guerraggio and Nastasi, 2005, 139.

[6] See Schultz, 1938, 64.

[7] See Stigler, 1954, 85.

[8] See Benini, 1907.

statistics came from Corrado Gini (1910), who, Stigler wrote again,[9] made use of a bivariate form to express demand as the function of the price of the same good or of a complementary or surrogate good.

However, economists-statisticians did not make only technical contributions: the economists-statisticians concurred to enliven the debate on the methods and visions of the research field, economic dynamics. To remember Benini only for his innovations in statistical tools may appear ungenerous, at least because we owe to Benini an original approach to economic studies defined by Gustavo Del Vecchio (1929, 958) as "economic sociology with statistical basis." Benini, in his turn, defined "inductive political economy" (1897, 1907, and 1908) as his research area; he was thinking of a new field where the use of multiple regressions and generally of statistics was grafted on an economic vision anchored to a "complex individual." Trying to clarify this concept, he wrote that "[the complex man] is not that abstract and extreme individual theorized by deductive economics" (1908, 17), but he is the continuously changing man who has a religious, political, and ethical characterization in addition to the economic one.

Benini carefully specified the notion of *complexity*, which was common in those years among scholars who adopted evolutionary approaches, but he stated it in a dynamic perspective and above all equipped it with what previously had been missed, an apparatus of statistical and quantitative tools. Benini (1908, 30) thought that "statisticians should not pay careful attention to observation only, they should be brilliant calculating persons," as would be requested of first econometricians after a while. It was greatly to his merit, as Bresciani Turroni wrote (1929, 994), that "an impulse to an approach showing an equilibrium between statistical and theoretical analyses has been given."

In the area of economic studies, at that time Costantino Bresciani-Turroni recognized the uncommon skill of combining a wide control of economic theory with a deep knowledge of statistical methodology. This talent culminated in the publication of *Le vicende del marco tedesco* (1931), which made him the point of reference for economic-statistical studies. The statistician Giorgio Mortara, reviewing *Le vicende*, appreciated the equilibrium between economics and statistics:

> The analysis of facts [and data] is not aimed at a mere description, but at searching causes, effects, interdependences. This attempt is never threatened by doctrinal biases. (Mortara 1932, 286)

Investigating economic fluctuations occurring in Germany between 1924 and 1931, Bresciani-Turroni revealed his doubts about the possibility of separating the trend from the cycle by

[9] See Stigler, 1954, 85.

interpolation. His preference was for a detailed analysis of the phenomenon. Bresciani gave a proof of what he thought by proposing a study of Germany's industrial fluctuations in which economic, psychological, and methodological aspects concerning both the cycle and the trend would be examined.

This view opened the way to conceive economic movements as a "complex" phenomenon, a perspective shared by statisticians and economists such as Mortara, Benini, Camillo Supino, Del Vecchio, Giovanni Demaria, Guglielmo Masci, and Marco Fanno. The economic cycle was not determined by a unique variable but was the outcome of many "little" oscillations. This view brings us back to A. Wagner (1892–94), to his idea of "complex causality,"[10] and, finally, to the belief that statistics may specify the many causes determining economic and social phenomena. Bresciani-Turroni's hypothesis according to which the cycle finds explanation in a multiplicity of causes acting jointly or separately illustrate this approach: such an approach could be defined *multiple disjunctive causality*.[11]

Finally, these authors did not use abstract terms such as "logical antecedent" or "subsequent" to express dynamic movement or change but used a "historical antecedent" or "empirical datum." This practice placed the relationship between cause and effect in the field of *factual relations*, far from merely theoretical formulations.[12]

2. Amoroso as the Forerunner of Econophysics Studies?

Pareto and Amoroso are both known by current econophysicians as early scholars of income distribution, an important research area of econophysics. Sometimes econophysicians have also stressed the wider Italian interest in distribution curves, quoting authors such as Felice Vinci and Raffaele D'Addario. This specific attention may be explained with Pareto's legacy but was also the consequence of a diffused belief well summed up by Gini in 1926:

> It was, however, important to establish indices having a descriptive value also for distributions quite different from the Gaussian ones. (1926, 714)

In the *Treatise on General Sociology*, Pareto (1916/1935) introduced the notion of individual and social heterogeneity, with the consequence that methods quite different from the Gaussian were requested. Paretian heterogeneity found expression in Amoroso's different research

[10] The term causality is here used with his meaning of *epistemological category*, which corresponds to *causal nexus*, that is, in a wider sense that *necessary relation between properties*. Although such a notion is employed in reference to cases in which changes are yielded by external events (as in the traditional idea of efficient cause); in a more general meaning, causality includes internal determinants.

[11] See Bunge, 1970, 145. If we assume C_1, C_2, …, C_n, as causes, E the effect, and ν the logical correlation, the disjunctive multiple causality may be written: $C_1 \vee C_2 \vee \ldots \vee Cn \rightarrow E$.

[12] A closer examination of the topics evoked in this section may be found in Tusset 2004, Chap. 2.

interests, in particular, his work on income distribution, the logistic curve of population (here not deepened), and, finally, his cycle theory.

In the 1925 article, *Ricerche intorno alla curva dei redditi*, Amoroso gave a clear view of the statistical perspective guiding his research on this topic, a feature that may be better understood by singling out the difference with the previous astonishing Pareto's wealth power law (1896–97/1964). Given Pareto's law in the first approximation as $N = A/x^{\alpha}$ where N represents the number of income units from the top above a given income limit x, A and α are constants, Amoroso re-wrote the income law as follows:

$$N = ce^{-\gamma(x-h)^{1/s}}(x-h)^{\alpha - s/s}, \qquad [1]$$

where c, h, γ, α, are positive constants; s is a constant that may be positive or negative, never nil.[13] Re-writing the income equation, Amoroso mostly disputed Pareto's famous statement affirming that inequality decreases with the growth of the average income. Differently, Amoroso stated that inequality may increase with average income because the monetary scale changes with income. Thus, the Paretian long tail is replaced by a distribution similar to a top: incomes rapidly rise and then gradually decrease following the Paretian long tail. Briefly, by substituting power-law distribution with an exponential distribution, Amoroso widened the level of the represented heterogeneity.

But the points of contact between Amoroso and econophysics are not limited to his income distribution theory. As we will try to argue, his conception of cycle is built on observed behaviors, as "habits," which became the pivot of his application of inertia principle to economics. Amoroso followed his teacher Maffeo Pantaleoni, who pointed out the social and individual "resistance to change" appearing when "economic dynamism … involves unknown goals" (Pantaleoni 1907, 217).[14]

Organizational aspects involve inertia in production. Consumers frequently show habits in their trade behaviors; all these are observed facts that may find expression in the cycle theory, maintaining an analogy with the physical world. In 1938, Amoroso wrote that the prices' dynamics constitutes the facts (1938, 111) on which theory should be grounded. While the movement of the different sectors is characterized by different delays, according to the then employed "economic barometers," the oscillation depends on the fact that each expansion of a variable is followed, besides its own inertia, by the reactions occurring in the other sectors. Prices and rate of interest

[13] Amoroso's 1925 contribution has been recognized as the first proposal of the generalized gamma (Γ) distribution, although in Anglo-Saxon literature this primacy was attributed to Stacy. See Kleiber and Kotz, 2003, 147.
[14] See also Tusset 2009, 278.

influence production whose changes affect expected prices. As we will see, bursts and falls characterize economic activity.

Amoroso imagined that the cycle is guided by price dynamics, in other words, by *mass speculation* producing a sort of *speculative bubble*. He wrote:

> … when speculation is general it becomes ruinous. This is the fundamental truth that explains why public opinion totally rejects it. After all people's conscience becomes aware of the danger implied by mass speculation supported by mass itself, people refuse it giving rise to a shift … The more heavy devastation is that occurring in the invisible field, that of trust, the more fragile spring of the whole economic system. (1938, 224)

Finally, this view grounded on acceleration and deceleration led Amoroso to conceive industrial production according to a thermodynamic perspective, where the potential energy is the economic potential of plants and kinetic energy is the concrete production of the system. The interest in this analogy rests on Amoroso's conclusions: dealing with the economy, a further type of energy must be considered, which finds it analogous in the thermic one and represents all dissipation and losses characterizing productive processes. Thus, the sum of potential energy and kinetic energy is not a constant.

3. A Dynamic Theory of Supply

To start the investigation of Amoroso's contributions to economic dynamics, it is useful to begin with one of the more original theories for that time, the theory of supply. Amoroso built his theory by adopting an approach that distanced him from neoclassical stances.

First in his most famous article, *La curva statica di offerta* (1930a), and then in *La dinamica dell'impresa* (1933), Amoroso tried to build a dynamic theory anchoring it to past, the *inertia*, but looking at future. First- and higher-order time derivatives of variables summarize the weight of the past or the weight of habits, in other words, the influence of those factors that hinder the achievement of static optimal output. Conclusion: in a dynamic economy, equilibrium must encompass further costs. The proof is of a behavioral type, not of a logical one. We will follow Amoroso's arguments.

Different from classical statements that affirm the equality between marginal cost and price, Amoroso concluded that, for any firm, the percent difference between price p and marginal cost m is directly proportional to the quotient among the amount of a firm's production q and the total amount of production Q, and indirectly proportional to demand elasticity ε (1930a, 10):

$$\frac{p-m}{p}=\frac{q}{\varepsilon Q}. \qquad [2]$$

Amoroso stated that demand elasticity is an "external quantity which varies with time" and, for this reason, would eliminate the "determinism" characterizing standard production formula (1933b, 448). In a dynamic perspective, firm's choices are conditioned by *forces of inertia* (*forze d'inerzia*) which affect decisions concerning production. Changing economy involves additional costs here represented by a dynamic coefficient n. Thus, assuming as constants ε, m, and n, and giving j the rate of interest, he wrote the following second-order equation that represents an effort to give a dynamic interpretation of the supply of the firm (1933b, 447):

$$\frac{p-\left(m+jn-nq'-nq''\right)}{p}=\frac{q}{\varepsilon Q}. \qquad [3]$$

The coefficient, n, represents technological and organizational features that certainly do not make easier any change in production. According to this, past may be both resource and a cost for the entrepreneur called to manage production change.

As stressed by Bordin (1935, 208 ff.), the past that Amoroso evoked is deterministically casted in the future. Past affects decisions of future production or, if we prefer, future is endogenously determined by past and present. The presence of the rate of interest in the [3] means that maximization involves some kind of discount of future variables. Thus, in this embryonic attempt to single out a way to dynamics the entrepreneur decides on the basis of his past and current experience. No exogenous shocks condition economic dynamics.

A further aspect concerns the empirical character of both demand and supply curves. Although Amoroso was a Pareto follower, this empirical bias was a legacy of the German historical school and Italian statistical doctrine. Quantities exchanged at given prices originate a *historical curve*, which connects the different equilibria resulting from the meeting of demand and supply curves. This bundle of curves tells us that prices change and consequently that supply changes. But the curves don't say anything about how the prices change and how supply reacts. Amoroso wrote:

> The demand curve – likewise for the supply curve – is not an experimental notion, or a synthesis of true facts, but it is rather an idea, or a logical scheme, or also an abstract category which does not allow us to predict future economic relationships. (1932, 424)

This means that it is not possible to draw separate supply and demand curves unless other assumptions are introduced. Thus, the starting point remains empirical or, better, experimental; that is, the historical curves from which, at given conditions, we can infer both the supplied and demanded quantities.

From Amoroso's viewpoint, the equilibrium between demand and supply is a potential or virtual one, that is to say, an ideal datum characterizing the interdependence between two variables, price and quantity, in a given moment or in an infinitesimal period. Thus, Amoroso's first aim is to find an equation that explains the interrelationship between price and quantity while setting aside mutual dependence. From such a perspective, Amoroso's dynamic theory does not show causal relationships but displays correlationships instead.

Amoroso tried to find a special equation that enabled him to express such a correlationship. The first attempt brought him to write a linear equation differential equation of supply presenting constant coefficients (Amoroso 1929, 77):

$$p' + hp = a + bq + cq' , \qquad [4]$$

where p is price of a given good q, p' and q' are the derivatives, a, b, c, and h are constants of which b, c, and h positive.

He based this first differential equation on Evans's equation that shows that the demand of a good depends not only on its price but also on the (growing or decreasing) trend of the price. Such a dependence on price and its movement can be expressed by the time derivative of price, $D(x) = f(p, dp/dt)$ (Evans 1930, Chap. IV). Apart from the fact that Evans included not only first-order derivatives but also higher-order ones, the time derivative simply represents the dynamic component of the equation. This theoretical approach is based on the hypothesis that there exists a linear relation between a variable and its change, but it is important to stress the early economic meaning Amoroso gave to time derivatives: they represented expected prices and expected quantities. It is true that *inertia* or *past* affect current choices, and that those time derivatives 'contain' the past, but such choices concern future.

4. "Classical" Dynamics

The interest in the economic analogous of mechanical inertia led Amoroso to develop his best-known approach to economic dynamics, grounded on the "calculus of variations" and well synthesized in his 1942 *Meccanica economica.*

The basic idea is that the economic movement is opposed by its own inertia assuming the character of *habits* in the case of consumption, of *organizational* and *physical constraints* for what concerns production.

Some insights into Pareto's view on the economic meaning of inertia are necessary. He developed some ideas in the second volume of *Cours* (1896–97, §586,[1] §928[2]) recognizing the existence of some "pains" to the consumer who is changing his habits. A change in consumption will increase the individual well-being of a value that takes into consideration the inertia, that is, the cost of change. If r is the good traded and then consumed by an individual, and m is a parameter expressing this cost, the inertia may be expressed as follows: $m(d^2r/dt^2)$. So, if φ is the increased utility without inertia, the final utility followed to the change will be: $\varphi - m(d^2r/dt^2)$. If the individual does not move from his initial position, the gain of utility is zero, in other words: $\varphi = m(d^2r/dt^2)$. The potential increase of utility is compensated by the existing inertia. Ignoring the weight of mechanical analogies, the idea of economic dynamic hypothesized by Pareto and then developed by Amoroso is entirely played on *economic change*, and by supposing that the latter has a cost for the individual.

Pareto wrote that the value of inertia is not known beside the nature itself of this function (1896–97, §586[1]), and this is true unless we change the perspective limiting ourselves to observing the phenomenon. This is what Amoroso did by inserting the inertia into his empirical analysis of economic fluctuation; briefly, he harmonized inertia in the economic barometers. The adoption of economic barometers was ascribable to the mechanics' influence on economics, particularly to the reception of Newton's third axiom, that of action/reaction. Actually, the naïve application of the third axiom inspired the building of the then-famous economic barometers such as the Harvard barometer proposed by Persons in 1916. The barometer is grounded on the assumption that two or three variables follow the same oscillatory movement presenting a time gap among them. These variables were generally number indexes representing aggregate behaviors on statistical bases.

According to Amoroso's view, the rational entrepreneur should maximize the following integral (1942, 158):

$$I = \int_{t_0}^{t_1} (Q - \Theta)dt\,, \qquad [5]$$

where Q is the production and Θ are the costs referred to dt. Assuming that x is the investment carried out by the firm, Amoroso wrote that the integral is maximized when:

$$\frac{\partial Q}{\partial x} - \frac{d}{dt}\left(\frac{\partial Q}{\partial x'}\right) = \frac{\partial \Theta}{\partial x}\,, \qquad [6]$$

where the second differential of the left part of the equation represents the inertia symbolizing organizational costs, including those due to physical features. Since $\frac{\partial Q}{\partial x'} < 0$, the inertia increases the costs reducing profits. In a few words, the entrepreneur maximizes his profit when marginal productivity is equal to the inertia at the margin. Something analogous occurs for consumer, who, dealing with habits, maximizes his utility when marginal utility equalizes inertia (habits) at the margin.

Applying this view to a period t, Amoroso showed that the maximization of both production and consumption is feasible by applying the *calculus of variation*, which well expressed the principle of minimum means in economics.[15] But, as Amoroso himself stressed in 1957, this dynamic did not depict the variability and instability of economic reality and did not give form to innovations boosting the economy. On the contrary, dynamics grounded in classical mechanics, such as that built on inertia, may represent a stationary or better steady growth economy.

5. An "Elastic" Dynamic Model

During the thirties, Amoroso worked out a way to the economic cycle that contained a different view of economic dynamics that partially changes also the notion of inertia itself. Based on Fisher's 1925 insights into the rate of change of prices, this model hinged on the role of the price elasticity of supply and for this was defined by Giuseppe Palomba, Amoroso's most loyal pupil, as an "elastic model" (1976, 120). Previously, Schumpeter had written that aggregative theories based on "elastic" methods "can easily be made dynamic by the introduction of lags and rate of change" (1939, 185). Amoroso's cycle theory was just grounded on rates of change and lags. In addition, we can say that the rate of change of economic variables, that is, accelerated movement, to make use of a physical concept, sometimes crash course, is the distinguishing mark of this approach. Rightly, Amoroso in 1940 evoked a supposed *principle of euphoria*, to set out the tendencies of economic variables and prices but also production: burst followed by fall. Later, Amoroso synthesized the dynamic model as follows:

> *a*) the utility function depends not only on the velocity of the flow of consumption goods, but also on its acceleration; *b*) production depends not only on the flow of productive factors but also on its acceleration. (1963, 18)

If the minimum means was the physical principle leading Amoroso pursuing Pareto's traces, the elastic law, *ut tension sic vis*, represented the guiding idea of his cycle theory. What came out was not only an interpretation of economic movements but rather a dynamic theory of economic phenomena. His theoretical bet was basically played on the following point: the aggregate phenomena are different from the individual ones; thus, aggregate variables may not follow an optimizing principle. They follow some rules or laws that can be induced by observation.

In the 1932 *Contributo alla teoria matematica della dinamica economica*, Amoroso sketched a first scheme that included two sectors: business (or speculation) and industry (or production). He showed how the interaction between the two sectors involves economic fluctuations and instability. Briefly, the scheme is the following.[16]

Amoroso wrote an *equation of industry*:

---

[15] See Pomini and Tusset 2009.

[16] We present here a slightly modified version of Palomba's account of Amoroso's 1932 theory. See Palomba, 1966, 593ff.

$$Q_{t+\omega} = a + m^2 P_t', \qquad [7]$$

where Q is the yielded wealth, a and m are constants, P is the price index, and ω is a given period of time. Thus, production depends on the time derivative of the prices.

He wrote also an *equation of business* or *speculation*:

$$P_t = b - n^2 Q_{t+\omega}', \qquad [8]$$

where b and n are constants.

It is clear that prices are grounded on future production decisions. Prices increase until expectations anticipate a growth in production. But prices cannot increase indefinitely. When prices are too high, the demand decreases, the speculative cycle shifts towards a sloping down phase.

By transforming and integrating, Amoroso obtained the following trigonometric system:[17]

$$\begin{aligned} P_t &= n(Hcos\lambda t + Ksen\lambda t) + b \\ Q_{t+\omega} &= v(Kcos\lambda t - Hsen\lambda t) + a \end{aligned} \qquad [9]$$

where H and K are constants and λ = 1/nv. Amoroso reached the conclusion that P and Q may have evolutionary characters, but surely they interact with each other. The variable leading the cycle is the price index in the speculative sector, since increasing expected prices may provoke investment in the production sector. But once the growth in production has saturated the market, the expectations change direction provoking sales in stock exchange and pushing prices towards a fall.

However, Amoroso was not satisfied with his cycle theory. He recognized that credit expansion may play an important role in determining investment in production and stressing the prices. By increasing circulation, the banking system may provoke an inflationary process. Then, the prices' rise may induce "forced savings." Then, with the scope of re-introducing the role of credit, in the 1935 "La dynamique de la circulation," published on *Econometrica*, Amoroso added an equation on the banking sector.

The speculation equation, now called the stock exchange equation (*équation de la Bourse*) became the following:

$$P_t = m - a_{11} P_t' - a_{12} j_{t+\omega}' - a_{13} Q_{t+\omega}', \qquad [10]$$

where a_{11}, a_{12}, a_{13}, and m are positive constants, j is the interest rate, and m is a parameter of monetary stock, M. Analogously, Amoroso wrote the banking equation:

$$j_t = m + a_{21} P_{t-\omega}' - a_{22} j_t' + Q_{t+\omega}', \qquad [11]$$

[17] Assuming $S_t = Q_{t+1}$ we can re-write the two equations in a system as follow $S_t = a + v^2 P_t' \quad P_t = b - n^2 S_t'$. Then we deduce $P_t = b - n^2 v^2 P_t''$ and putting λ = 1/nv we have: $P_t'' = -\lambda^2 P_t + \lambda b$. Integrating this second-order differential equation by the Eulero-MacLaurin formula, we obtain $P_t = Acos\lambda t + Bsen\lambda t + b$ and $S_t = Ccos\lambda t + Dsen\lambda t + a$, where A, B, C, and D are constants. From these, it is easy to obtain the equation system [9] quoted in the text (see Palomba, 1966, 594–95).

and the production or industry equation:

$$Q_t = m + a_{31} P'_{t-1} - a_{33} j'_{t-1} - a_{33} Q'_t \, . \qquad [12]$$

The equation system now shows the influence of the interest rate on both production and price expectations. Moreover, changes in the monetary stock directly affect the three current variables. Thus, in Amoroso's view, such an equation system expresses the synthesis between quantitative theory and the cyclical movement (1935, 407).

The system of equations is cyclical because it includes first-order differential equations with constant coefficients that admit periodic integrals. The cyclical motion of both the price and production is endogenous, because the motion is explained without resorting to exogenous shocks. In this regard, Amoroso anticipated, on a microeconomic level, subsequent work by Kaldor and Hicks. In fact, Amoroso founded his cycle theory on the intrinsic dynamics of the system itself: "... a movement, in a certain way, gives rise to another opposing motion: and the latter, at its turn, causes a change with antithetic direction, and so on" (1932, 433).

A simple qualitative analysis, based on data more than equations, pushes us to say that prices affect both banking activity and production choices.[18] In the latter, the interest rate also weighs. But, at the same time, forecasts concerning production guide speculative actions. Since 1932, Amoroso was conscious of the revolutionary potential linked to the introduction of speculative forces in economics. In other words, he knew that price and banking forecasts cannot be referred to natural or mechanical laws. Consequently, he knew that speculative forces could produce market "strains," in economic terms, market crises, or recessions.

We say that the increase in the investment volume, $\Delta k/k$, may push up prices, $\Delta p/p$. But the investments also increase according to interest rate changes, $\Delta j/j$. It is worthwhile to outline the outcome of such a system: price changes affect credit and production; but production, also influenced by disposable credit, determines speculative forecasts. In some sense, these relations bring us to think that the cycle is a sequence of speculative bubbles: prices increase on the basis of expectations. The latter change when production is expected to reach its potential level or when market is saturated. The bubble, once blown up, leads sooner or later to a depression in the real economy. Finally, we agree with Bordin when he affirmed that, although correlations had replaced causal relations, business action could assume an independent direction (Bordin, 1935, 835).

[18] Now, the three linear first-order differential equations must be analyzed in depth. First, the economic system may present a cyclical motion as the outcome of the interaction of nine different types of reactions. Three are reactions of inertia (marked by t), which arise in any sector; such reactions appear as correlations among the current value of production, prices, and the interest rate, on the one hand, and expected variations of such variables respectively, on the other (Q and Q', P and P', j and j'). Sure enough, we have six induced actions characterizing the industrial sector, which can be of the mechanical type. Occurring with a delay, they are marked by $t - \omega$. And the "speculative" actions based on forecasts, which, occurring in advance, are marked by $t + \omega$. The latter characterize the stock exchange sector. Finally, those of the banking system are of the mechanical and speculative type.

Euphoria and speculative bubbles are evoked by Amoroso in deepening the passage from micro- to macro-economic analysis, a point that Amoroso did not clarify in a satisfying way. Individual behaviors, when they are aggregated or considered as mass phenomena, can generate crises or perturbations. We can ask if Amoroso thought of a simple addition process or if he meant something else. Afterwards (1943a), Amoroso introduced a distinction between two types of inertia: first, the known individual inertia and, second, the inertia of the market. Both types of resistance concern market choices, but only the aggregate inertia can generate the rigidity of the market or the stickiness of variables. Amoroso stated that market failures are generated by joint behaviors. This means that Amoroso did not treat aggregate variables as the sum of individual ones. In addition, only the analysis of aggregates may explain those mass phenomena as imitation and euphoria that appear illogical by definition.

It is true that variables expressing inertia represent a general tendency of the economic system to maintain a specific situation, even when the originating causes disappear. However, we can say that the current value depends on past values from formal or statistical viewpoints, without any causal relation. Bordin (1935, 378) wrote that this is a "pure description of the phenomenon, and not its interpretation," anticipating a true econophysical approach. Amoroso's cycle theory had indeed lost any deterministic character.

A particularity of this scheme is represented by the presence of an evolutionary component that originates with the integration of the three equations. Similarly, with the schemes proposed by Frisch, Kalecki, and others, the cycle was grafted on a long-term trend. The latter, however, is not an economic explanation since it arises from specific integrals.

In fact, the integrals present the following form (1935, 409):

$$P = Ae^{rt} + Be^{st} cos\left[\frac{2\pi t}{\sigma} + z\right], \qquad [13]$$

where the first term of the right side is the evolutionary component and the second is the cyclical one. P is a price index, A and r are, respectively, the extent and the intensity of evolutionary movement, B, σ and h are, respectively, the extent, the period and the phase of the cyclical movement, z is a constant, and finally, s is a constant that dims the oscillations.

Amoroso judged the 1935 treatment as definitive. Such a conclusion certainly represented a turning point along the path of his theory, since he no longer denied the cyclical scheme here presented. However, such a conclusion also constituted a starting point. At first, Amoroso followed the way of economic planning along the line of classical mechanics. But later, he understood that "elastic" analogies may say something else of some importance. This second line of research was only sketched by Amoroso, also as spin-off of his interest in thermodynamic analogies.

6. The "Thermodynamic Approach"

Although included in his *Meccanica economica*, Amoroso's thermodynamic approach may be better understood if it is read in light of his cycle theory, particularly the "lagging potential" that the approach implies. Actually, the increase in prices yields delayed production growth; similarly, banks' credit constitutes a potential for investments, and finally, each industrial system presents productive potential. Amoroso's *thermodynamic approach* is grounded on the analogy between physical transformations, which are of an energetic type, and economic transformations, which involve value.

Assuming that an industry produces q employing productive factors x_s where $s = 1, 2, \ldots, n$, with price, p_s, M_s indicates the sum of x_s employable in the plant. So the investable value M will be the following:

$$M = \sum_{s=1}^{n} p_s M_s \,. \qquad [14]$$

If the plant is used for $X_s = x_{qs}$ the reserve production capacity V will be:

$$V = \sum_{q,s} p_s \left(M_s - x_{qs} \right). \qquad [15]$$

Thus, V may be considered the *potential energy* or *economic potential* of the plant or system.[19] Being the derivative of V:

$$dV = -\sum_{q,s} p_s dx_{qs} \text{ which can be written } dV = -d\Theta \,, \qquad [16]$$

where $d\Theta$ is the marginal cost.

That is, the economic potential is equal to the marginal cost with the opposite sign. Finally, if Q is the value of production (*kinetic energy*), being in equilibrium $dQ = d\Theta$, the thermodynamic interpretation of economics proposed by Amoroso leads to the following:

$$\frac{dQ}{dt} + \frac{dV}{dt} = 0 \,. \qquad [17]$$

Assuming $j = 0$, which means that the prices p_s are constant and the productive process is stated to be instantaneous (1942, 161), Equation [17] involves that production is extended up to the point where the differential of potential dV is equal to the differential of the value of the product dQ with opposite sign (Amoroso 1940, 7). Consequently, the marginal principle may be interpreted as the energy conservation principle (1942, 162). In this way, the economic system was conceived as a *conservative system*, that is, $Q + V = const$.

[19] Amoroso first proposed the notion of *economic potential* in the 1924 article *Meccanica economica*, with the meaning of Pareto's index function of an economic system (1924, 54).

But the analogies between the physical and economic world end here. Part of the "economic energy" will be dispersed or dissipated as the effect of previously existing interest on capital and rent, of time-consuming productive processes, and, we add, as a consequence of speculative forecasts on production. If Ω represents the dispersion, the analogous of thermic energy here expressed as monetary energy, Equation [17] becomes the following:

$$\frac{dQ}{dt}+\frac{dV}{dt}+\frac{d\Omega}{dt}=0\,. \qquad [18]$$

And, more generally, $Q+V+\Omega=const.$ On these bases, Amoroso concluded the following:

> … the *transformation of value which occurs in the dynamics of the productive process can be likened to the transformation which is effected in a mechanical process* and like the latter is governed by a principle analogous to that of the *conservation of energy*, with this fundamental difference: that the conservation of energy in the mechanical process represents a *natural law* which teaches us how certain facts occur, while, on the contrary, the transformation of value which is effected in the productive process represent a *rule of conduct*, which tells us how the facts occur, *if the conduct of individual is affected by a criterion of rationality*. (1940, 11, italics in the original)

Even if Amoroso did not explicitly state the following conclusion, it is clear that, according to Equation [18], the economic system is now conceived as a *dissipative*, not a *conservative*, system. At that point, the principle of least action assumes, when applied to economics, a different meaning from that resulting when it is applied to "nature" or physical phenomena.

7. Concluding Remarks

If Amoroso is mainly remembered for his attempts to dynamize Pareto's general equilibrium on deductive bases, his less known cycle theory certainly represented an experiment of 'observational' dynamics, that is, of dynamic theory grounded not on causal relations but on a pure description. Inertia, habits do not cause present and future. They are 'facts' on which future is built, without any deterministic prescription. This seems a very different approach from Amoroso's classical dynamics including 'calculus of variations' and the mathematical theory of economic program (see Pomini and Tusset 2009).

This cycle is also plunged in a speculative environment as if bubbles would determine choices in real economy. Prices are speculative or expected ones and quantities follow prices more than demand. Amoroso knew that price and banking forecasts cannot be referred to mechanical laws and that speculative forces could bring to market "strains," that is, to market crises, or recessions. Expectations on prices play a crucial role in determining economic movement and cycle.

This point undermines the celebrated (also by himself) Amoroso's mechanical faith. Really the experimental or observational character shapes the whole cycle. Certainly, mathematical tools may be drawn from physics, but this is a common trait of almost all the early dynamic theories.

References

Amoroso L. (1921), *Lezioni di economia matematica*, Bologna, Zanichelli.

Amoroso L. (1924), Meccanica economica, *Giornale degli economisti e Rivista di statistica*, 64, 45-54.

Amoroso L. (1925), Ricerche intorno alla curva dei redditi, *Annali di matematica pura e applicata,* 123-57.

Amoroso L. (1929), "Le equazioni differenziali della dinamica economica", *Giornale degli economisti e rivista di statistica*, 69-79.

Amoroso L. (1930a), "La curva statica di offerta", *Giornale degli economisti e rivista di statistica*, 1930, N. 1, 1-26.

Amoroso L. (1930b), "Introno alla determinazione empirica delle leggi della domanda e dell'offerta", *Giornale degli economisti e rivista di statistica*, 941-44.

Amoroso L. (1932), "Contributo alla teoria matematica della dinamica economica" in *Nuova collana degli economisti*, Vol. V, Torino, Utet, 419-40

Amoroso L. (1933a), *La dinamica dei prezzi*, G.U.F., Roma, 1933. Reprinted L. Amoroso, *Ciclo, circolazione, politica monetaria*, (ed. by L. Venturi), Torino, Utet-Bancaria Editrice, 1999.

Amoroso L. (1933b), "La dinamica dell'impresa", *Rivista italiana di statistica, economia e finanza*, settembre, 442-51.

Amoroso L. (1935), La dynamique de la circulation, *Econometrica,* 3(4), 400-410. First publication, La dinamica della circolazione", *Rivista italiana di statistica, economia e finanza*, 13(5), 823-36.

Amoroso L. (1938), *Principii di economia corporativa*. Bologna, Zanichelli.

Amoroso L. (1939), La teoria matematica del programma economico, in AA.VV. *Cournot nella economia e nella filosofia*, Padova, Cedam, 125-44

Amoroso L. (1940), The transformation of value in the productive process, *Econometrica*, 8(1) 1-11.

Amoroso L. (1942), *Meccanica economica*. Corsi del Reale Istituto Nazionale di Alta Matematica, Bari, Macrì.

Amoroso L. (1943a), *Lezioni di economica*, Bologna, Zuffi.

Amoroso L. (1943b), Meccanica economica e statistica matematica, in AA.VV., *Studi in memoria di Guglielmo Masci*, Milano Giuffrè.

Amoroso L. (1947), L'indice di concentrazione dei redditi secondo Pareto, *Rivista italiana di demografia e statistica*, ottobre, 134-39.

Amoroso L. (1948), Pareto matematico ed economista, in Accademia Nazionale dei Lincei, *Vilfredo Pareto economista e sociologo*, Roma, Bardi, n.10, 5-34.

Amoroso L. (1957), Modelli meccanici e modelli economici, *Studi economici*, 271-82.

Amoroso L. (1963), Modelli economici e modelli statistici, in AA.VV., *Studi di economia finanza e statistica in onore di Gustavo Del Vecchio*, Padova, Cedam, 2 voll., 1-19.

Benini R. (1897), Di alcune curve descritte da fenomeni economici aventi relazioni con la curva del reddito e con quella dei patrimoni, *Giornale degli Economisti*.

Benini R. (1907), Sull'uso delle formule empiriche nell'economia applicata *Giornale degli Economisti*, 35, 1053-63.

Benini R. (1908), Una possibile creazione del metodo statistico «L'economia politica induttiva», *Giornale degli Economisti*.

Boianovsky M. and Tarascio J.V. (1998), Mechanical inertia and economic dynamics: Pareto on business cycles, *Journal of the History of Economic Thought*, 20(1), 5-25

Bordin A. (1934), *Lezioni di economia politica e corporativa*. *La statica*. Vol. I, Padova, Cedam.

Bordin, A. (1935), Il significato di alcune moderne teorie matematiche di dinamica economica, *Giornale degli economisti e rivista di statistica*, 75, first part: 161-210; second part: 369-421; third part: 580-611.

Born M. (1939), Cause, purpose and economy in natural laws, in *Physics in My Generation*, London and New York, Pergamon Press, 1956, 55-79.
Bresciani-Turroni C. (1929), Alcuni effetti economici dei prestiti esteri in Germania negli anni 1924-1929, *Giornale degli Economisti e Rivista di Statistica*, 994-1067.
Bresciani-Turroni C. (1931), Le vicende del marco tedesco, *Annali di Economia*, IX.
Crooks G.E. (2007), The Amoroso Distribution, *Technical Note* 003v1.
Del Vecchio G. (1929), Le teorie economiche di Rodolfo Benini, *Giornale degli Economisti e Rivista di Statistica*, 957-66.
De Pietri-Tonelli A. (1921), *Lezioni di Scienza economica razionale e sperimentale*, Rovigo, Industrie Grafiche Italiane.
De Pietri-Tonelli A. (1931), *Corso di politica economica*. *Introduzione*, Padova, Cedam 1921.
Düring B., Matthes D., and Toscani G. (2008), A Boltzmann-type approach to the formation of wealth distribution curves, *Social Science Research Network*, 1-57.
Evans G.C. (1930), *Mathematical Introduction to Economics*, New York, McGraw Hill.
Fisher I. (1925), Our Unstable Dollar and the So-called Business Cycle, *Journal of American Statistical Association*, June, 179-202.
Gini C. (1926), The Contributions of Italy to Modern Statistical Methods, *Journal of the Royal Statistical Society*, 89(4), 703-24.
Hicks, J. (1939), *Value and Capital*, Oxford, Clarendon Press.
Keppler J.H. (1994), "Luigi Amoroso (1886-1965): Mathematical Economist, Italian Corporatist", *History of Political Economy*, 26(4), 549-611.
Keynes J.M. (1930), *A Treatise on Money*, I. *The Pure Theory of Money*, London, Macmillan.
Kleiber C. and Kotz S. (2003), *Statistical Size Distributions in Economics and Actuarial Sciences*, Hoboken, Wiley.
La Volpe G. (1938), *Ricerche di dinamica economica corporativa*, Padova, Cedam.
Majorana E. (1942), Il valore delle leggi statistiche nella fisica e nelle scienze sociali, *Scientia* 31, 58-66.
Mantegna R.N. and Stanley E.H. (2000), *An Introduction to Econophysics. Correlations and Complexity in Finance*, Cambridge, Cambridge University Press.
Matthews R.C.O. (1984), Darwinism and Economic Change in D.A. Collard. N.H. Dimsdale, C.L. Gilbert, D.R. Helm, D.R. Scott. A.K. Sen (eds.), *Economic Theory and Hicksian Themas*, Oxford, Clarendon Press, 91-117.
Mortara G. (1932), Recensione a C. Bresciani-Turroni, Le vicende del marco tedesco, *GdE e RdS*, 286.
Palomba, G. (1966), *Fisica economica.* Napoli, Giannini.
Palomba, G. (1976), *Saggi critici*. Roma, Libreria Eredi V. Veschi.
Pantaleoni M. (1907), Una visione cinematografica del progresso della scienza economica. In *Scritti vari di economia*. Vol. II, *Erotemi*. Bari, Laterza.
Pareto V. (1896-97/1964), *Cours d'économie politique*. Genève: Droz, Tome I et II. Nouvelle édition par G.H. Bousquet et G. Busino.
Pareto V. (1916/1935), *A Treatise on General Sociology*. *The Mind and Society*. 2 Vols., New York, Dover Publications. Italian edition 1916.
Pomini M. and Tusset G.(2009), Habits and expectations: Dynamic general equilibrium in the Italian Paretian School, *History of Political Economy,* 41(2), 311-42.
Prevost J-G. (2002) Genèse particulière d'une science des nombres. L'automatisation de la statistique en Italie entre 1900 et 1914, *Actes de la recherché en sciences socials*, 141-42, 98-109.
Schumpeter J.A. (1939), *Business Cycles*. *Volume I*, New York and London, McGraw-Hill.
Stigler J.P. (1954), The Early History of Empirical Studies of Consumer, *The Journal of Political Economy*, LXII, 95-113.
Tusset G. (2004), *La teoria dinamica nel pensiero economic italiano (1890-1940)*, Firenze, Polistampa.
Tusset G. (2009), The Italian contribution to early economic dynamics, *The European Journal of the History of Economic Thought*, 16(2), 267-300.
Wagner A. (1892-94), *Grundlegung der politischen Ökonomie*, Leipzig, 3 Auf. Tr. franc. parz. *Les fondements de l'Economie politique*, Paris, Giard & Brière, 1904