Financial constraints and unemployment equilibrium
Giovanni Cesaroni* and Marcello Messori**

Abstract

The aim of this paper is to show that a robust determination of unemployment equilibria can be based on the integration of credit rationing into a general equilibrium model. We first review some of the traditional macroeconomic models of the neoclassical synthesis in order to stress that these models will be a coherent solution to Keynes’s analysis of unemployment, if the relaxation of the general equilibrium framework is based solely on exogenous price and quantity constraints. We then maintain that the consequent determination of unemployment equilibria is analytically fragile, and we show that the problems bequeathed by the IS/LM models are only partially solved by the strand of the new Keynesian economics based on market imperfections and endogenous rigidities. In order to overcome these problems we refer to credit rationing. In particular, we build a simple general equilibrium model in which prices are – in principle – perfectly flexible and credit rationing implies unemployment equilibria. Apart from the crucial role played by the credit market, our model is very similar to that developed by the neoclassical synthesis.

Jel Classification: E12, E44, G21

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1. Introduction

Credit rationing, once a major issue of debate in the literature on banking, has moved out of the spotlight since the mid-1990s. If interest in the matter has waned, it is not because inquiry reached an analytical dead end or, on the contrary, achieved definitive results. Although the numerous attempts made in the 1960s and 1970s to explain positive excesses in credit demand at the equilibrium interest rate proved unfruitful, at the turn of the 1980s the fundamental works of Keeton (1979, ch. 3) and Stiglitz and Weiss (1981) provided a rigorous, if restrictive, justification of equilibrium credit rationing in the presence of \textit{ex ante} asymmetries of information between a lending bank and a subset of borrowers with projects entailing different but, for the bank, indistinguishable degrees of risk. These contributions marked a watershed, ushering in a second, fruitful phase of research testing the robustness of the results reached under less restrictive assumptions than those required by Stiglitz and Weiss. The outcome was a weakening of the possibility of rationing. Further inquiry did not ensue, however, and the issue of credit rationing gradually moved to the backburner both in the theoretical literature and as a policy problem.

A first objective of this paper is to show that the lapse of interest in credit rationing has carried high costs for the study of unemployment equilibria, blocking a promising way to solve the problems opened by the legacy of the “neoclassical synthesis”. Until now economic theory has been unable to construct robust aggregate general models with unemployment, and has fallen back on exogenous price and quantity constraints. By contrast, the integration of credit rationing into a general model could determine unemployment equilibria based on endogenous quantity
The second objective of our paper is to pursue this integration. Accordingly, we propose a simple model in which we obtain credit rationing and unemployment equilibria.

The rest of the paper is organized as follows. In Section 2 we review some of the traditional macroeconomic models of the neoclassical synthesis in order to show that the analytical fragility of the unemployment equilibrium in general models stems from its being based on exogenous price and quantity constraints. The recent contributions of the new Keynesian economics, based on imperfect markets and endogenous rigidities, solve only a part of the problems bequeathed by the neoclassical synthesis. Accordingly, in Section 3 we revisit the salient points of the debate on credit rationing, which introduces endogenous quantity constraints into the financial markets without other exogenous rigidities. In Section 4 we proceed to construct a simple aggregate general equilibrium model with credit rationing and demonstrate its relevance to the unemployment problem at hand. As we show in Section 5, in our model unemployment equilibria can be reached that do not depend on the assumption of limited downward flexibility of real wages. In Section 6 we summarize our results and discuss some implications of our model.

2. Price constraints, quantity constraints and unemployment

To begin to clarify the possible link between credit rationing and unemployment equilibria, it is useful to review several stages of the vicissitudes of Keynesian economics and, in particular, the neoclassical synthesis. It is generally acknowledged that Keynes (1936) did not provide adequate justification of the links between flexible nominal wages, changes in the expectations of entrepreneurs and wealth owners, and the persistence of involuntary unemployment. In chapter 19 of the General Theory he drops the previous hypothesis of rigid money wages in order to show that their downward flexibility does not eliminate involuntary unemployment, since it can have a

1 In this paper we use the expression “endogenous quantity constraint” to denote constraints that arise from economic agents’ rational choices and impede market clearing, i.e. the adjustment of actual quantities to notional ones.
negative impact on the expectations of entrepreneurs and wealth owners. The point is that, not having specified the determinants of expectations, Keynes is forced to justify their negative change by introducing assumptions which, though perhaps empirically plausible, remain analytically *ad hoc*.

As is well known, this unsolved problem of Keynes’s analysis was at the center of the manifold versions of the neoclassical synthesis. In particular, Hicks (1937) shows that flexible prices for labor and goods do not lead to a full employment equilibrium only if there are downward rigidities in the interest rates on financial assets (the so-called liquidity trap) blocking the adjustments prompted by the fall in the demand for money due to the transactions-motive (the so-called Keynes effect). The weak points, evidenced by Hicks’s Keynesian equilibria, stimulated the analysis by Modigliani (1944), which identifies rigidities in real and money wages as one of the necessary conditions for obtaining unemployment equilibria. Modigliani’s model is the most robust, albeit still unsatisfactory, version of the neoclassical synthesis.

To introduce the problem of the unemployment equilibrium, let us take a standard Keynesian model. Assume as exogenously given: the money supply \( M_s \), the labor supply \( N_s \), the average propensity to consume \( c \) and the marginal efficiency of capital \( mec \); assume, too, that the marginal productivity of labor is decreasing. Under simplified hypotheses, we have:

\[
\begin{align*}
(2.1.) & \quad M_d = M_1(Y_d) + M_2(r) \quad \text{with } dM_1/dY_d \geq 0, \ dM_2/dr \leq 0 \\
(2.2.) & \quad M_s = M_d \\
(2.3.) & \quad y_d = I(mec, r) + C \quad \text{with } dI/dr < 0 \\
(2.4.) & \quad C = c\ y_s \quad \text{with } 0 < c \leq 1 \\
(2.5.) & \quad y_d = y_s \\
(2.6.) & \quad y_s = f(N_d) \quad \text{with } dy_y/dN_d \geq 0 \\
(2.7.) & \quad w/p = dy_y/dN_d \quad \text{with } d^2y_y/dN_d^2 < 0
\end{align*}
\]
(2.8.) \[ N_d - U = N_d \] with \( U \geq 0 \)

(2.9.) \[ Y_d = p y_d \equiv Y \]

In a first stage, let the money wage rate \((w)\) also be exogenously set. The system (2.1.)–(2.9.) is able to determine: the demand for money \((M_d)\), divided into the demand for transactions-motive \((M_1)\) and for precautionary-motive and speculative-motive \((M_2)\), the interest rate that ensure equilibrium in the money market \((r)\), the aggregate demand for investment \((I)\) on the part of firms, the aggregate demand for consumption \((C)\), the levels of aggregate demand \((y_d)\) and supply \((y_s)\) that ensure equilibrium in the goods market, the demand for labor \((N_d)\), the amount of unemployment \((U)\), the equilibrium general price level \((p)\) and the consequent monetary value of effective demand and equilibrium income \((Y_d, e, Y)\).

According to Keynes (1936, ch. 18), the equations system constituting the macroeconomic model is decomposable: given the money supply \((M_s)\), the equilibrium in the money market determines \(r^*\), that is the equilibrium interest rate; on this basis, and given \(mec\) and \(c\), the equilibrium level of real income \((y^*)\) is determined by means of the income multiplier and hence the level of employment by means of the production function, \(f(N_d)\); since the level of money wages \((w)\) has been assumed as given, \(y^*\) and the function of the marginal productivity of labor determine both the optimal demand for labor and the equilibrium price level \((p^*)\); given \(y^*\) and \(p^*\), lastly, the equilibrium level of monetary income \((Y^*)\) is determined. Robertson (1936), Hicks (1937; see also Keynes 1973, pp.79-83) and other representatives of the neoclassical synthesis find that Keynes’s affirmation is unfounded because it does not consider the feedback effect of (2.5.) and (2.9.) on (2.1.) — i.e. the equilibrium monetary income’s influence on the demand for money due to the transactions-motive. This renders equations (2.1.)-(2.8.) interdependent (see also Messori 1991).

The implication commonly drawn from this observation of Robertson and Hicks is the following: given flexible money wages, it is precisely the interdependence among the variables of the system that undermines the Keynesian unemployment equilibrium. Let us pursue a proof \textit{ab absurdo} by
supposing that $U > 0$ and by dropping the previous, provisional assumption of predetermined $w$. To render money wages endogenous, an equation must be added to the system (2.1.)-(2.9.):

\begin{equation}
(2.10.) \quad w = w(u) \quad \text{with} \quad u = U/N; \quad dw/du < 0
\end{equation}

Given (2.10.), according to the standard approach $U > 0$ implies that the system is not in general equilibrium as there is a negative excess demand in the labor market. On the basis of (2.7.), a downward adjustment of money wages can offset this excess by augmenting $N_d$; but given (2.6.), such an adjustment generates a negative aggregate excess demand in the goods market (see (2.5.)), entailing a downward adjustment in the general price level, $p$ (see (2.9.)). On the basis of (2.1.), the fall in $p$ causes a reduction in $M_1$, which will not involve disequilibria in the money market only if there is an adequate increase in $M_2$ in response to a decrease in $r$ (see also (2.2.)). Given (2.3.), this decrease has expansionary effects on $I$ and on aggregate demand. This complex of simultaneous adjustments in prices and quantities ends when the system reaches an equilibrium in all markets, including the labor market. From the interpretation in question, it follows that the price adjustments in the labor, goods and money markets eliminate every kind of unemployment.\(^2\) The assumption of a persistent $U > 0$ must be rejected because absurd.

However, Hicks (1937) introduces new hypotheses to safeguard Keynes’s results. He interrupts the chain of price adjustments by imposing the condition that — owing to the liquidity trap created by the pessimistic and exogenous expectations of wealth owners — $r$ does not fall and hence does not stimulate an increase in investment and the consequent increase in aggregate demand for goods. That is, in place of (2.1.), we have:

\begin{equation}
(2.1.\text{bis}) \quad M_d = M_1(Y_d) + M_2(r, r_{\text{min}}) \quad \text{with} \quad dM_1/dY_d \geq 0,
\end{equation}

\(^2\) Our thesis is that the adjustment process described is in itself unable to ensure a full-employment equilibrium. It is certainly possible to construct a sequence of adjustments leading to such an equilibrium, but it would be a case arising from specific relations among the variables. This suggests that money wage rigidity could be the consequence, and not the cause, of the system’s inability to reach a full-employment equilibrium. Here we have chosen to share the traditional interpretation for two reasons: first, because it is typical of the neoclassical synthesis, and, second, because, as Robertson (1951) and Modigliani (1944) emphasize (see below), a monetary policy that adjusts the quantity of money in terms of wage units can guide the economy toward its ‘natural’ equilibrium position.
\[ \frac{dM_2}{dr} < 0 \mid r > r_{\text{min}}; \quad \frac{dM_2}{dr} = -\infty \mid r = r_{\text{min}}. \]

where \( r_{\text{min}} \) is the minimum level, which constrains the downward adjustments in \( r \); being set by the expectations of wealth owners, \( r_{\text{min}} \) is “conventional” and therefore, in the present analytical context, exogenously given.

Assume that the constraint placed by (2.1.bis) is binding (liquidity trap) with \( U > 0 \). If, with a fall in \( w \), aggregate supply positioned itself at the level compatible with full employment, this would render the negative excess demand in the goods market persistent. Thus, (2.5.) and (2.6.) constrain (2.7.): firms’ decisions, geared to maximizing their expected profits, reproduce the situation of involuntary unemployment even when money wages are flexible downward.

Patinkin (1948; see also 1965) exposes the theoretical weakness of this conclusion. He proves that if \( w \) is flexible downward, the so-called Pigou effect (or wealth effect) can be combined with the Keynes effect, thereby ensuring general equilibrium and full employment even when \( r \) is rigid below a given threshold. To obtain this result, it is sufficient to replace equation (2.4.) with:

\[ (2.4.\text{bis}) \quad C = c_1 y_s + c_2 \frac{E}{p} \quad \text{with} \quad 0 < c_1, c_2 \leq 1; \quad \frac{dC}{dE/p} > 0 \]

where: \( E/p \) is the amount of wealth in real terms and \( E \) is exogenously given.

In the system (2.1.bis), (2.2.)-(2.3.), (2.4.bis), (2.5.)-(2.9.) and (2.10.) it is still true that the decrease in the general price level does not have positive effects on investment, owing to the liquidity trap; however, an adequate decrease in the general price level increases the amount of “real” wealth and thus generates the necessary increase in aggregate demand for goods through increments in consumption demand. Consequently, the fall in money wages triggers price adjustments that can lead to a full-employment equilibrium in the new system of equations as well.
Here, we need not dwell on the reasons why Patinkin’s (1948) analysis is open to criticism on empirical grounds. Rather, the important point is that we draw the conclusion that the neoclassical synthesis offers only one model able to arrive at a well-founded unemployment equilibrium, namely Modigliani’s (1944). The latter assumes that nominal wages are rigid downward; thus, *ceteris paribus*, in the labor market any negative excess demand cannot be offset by downward adjustments in money wages and, hence, in the general equilibrium price level. Returning to the original system of equations (2.1.)-(2.9.), we find that (2.7.) can be better specified as:

\[(2.7.\text{bis}) \quad \frac{w}{p} = \frac{dy_s}{dN_d} \quad \text{with} \quad w = w_{\text{min}}\]

It is legitimate to assume that \(U > 0\) when \(w = w_{\text{min}}\). If this is so, in order to obtain a full employment general equilibrium it is necessary to drop the *ceteris paribus* clause. In particular, it is necessary for there to be an expansionary monetary policy such that, in place of (2.2.), we have:

\[(2.2.\text{bis}) \quad M^*_s = M_d(r_{\text{FE}})\]

where \(M^*_s\) is the quantity of money, exogenously determined by the monetary authorities, that ensures the interest rate level \((r_{\text{FE}})\) able to determine the investment decisions and hence the amount of aggregate demand and the general price level \((p_{\text{FE}})\) that, for a given \(w_{\text{min}}\), determine a real wage rate \((w_{\text{min}}/p_{\text{FE}})\) that will maximize firms’ expected profits at full-employment income – i.e. that will satisfy (2.3.) and (2.5.) as well as (2.6.) and (2.7.) for \(y_{\text{FE}}\).

Modigliani (1944) thus builds a model where there are two necessary conditions to have an economic system in an unemployment equilibrium: nominal rigidities \((w = w_{\text{min}})\) in the labor market and a quantity constraint \((M_s = M < M^*_s)\) in the money market that exogenously sets the quantity of money in terms of wage units at a level too low to be compatible with \(y_{\text{FE}}\). Through adjustments in both the “real” and the monetary variables, these two conditions determine a real wage rate

\(^3\) Without referring to the abundant literature on the subject, suffice it to recall that a fall in prices worsens the monetary position of debtors, and since firms are the economic system’s typical debtors this is likely to lead to a retrenchment in investment. Empirically, in other words, the positive impact of the wealth effect on aggregate demand for consumption...
competitive balance on the money market (w_{min}/p)* | p^*<p^{FE}) higher than the corresponding level of full-employment demand in the labor market.\footnote{The link is indirect, in that the quantity theory of money does not hold and the monetary variables have real effects. Note too that, following the terminology of the new Keynesian economics (see below), from now on we will speak of “nominal rigidities” in connection with absolute prices and “real rigidities” in connection with relative prices.} It should be noted that the nominal rigidities in the labor market and quantity constraints in the money supply safeguard Keynes’ results, but at the price of drastically weakening their implications. Keynes aspired to reducing the standard full employment equilibrium to a particular case of his “general theory”. By contrast, Modigliani’s neoclassical synthesis underscores that it is the unemployment equilibria that are analytically fragile – so fragile that exogenous rigidities and constraints hindering the “normal” working of the markets must be introduced into the model. The theoretical problem that Modigliani bequeaths to the followers of Keynes’s theory is therefore clear: to render endogenous the rigidities and constraints that the Keynes’s model requires.

This leads us to establish a close link between Modigliani’s approach and the recent models of the new Keynesian economics with market imperfections and endogenous rigidities. Besides assuming some form of imperfect competition in the goods market and sometimes in other markets as well, these models seek to microfound both the nominal rigidities, by introducing “quasi rational” behavior or “small menu costs” in the goods market, and the real rigidities, by introducing “efficiency wages” or other analogous hypotheses in the labor market (see Akerlof and Yellen 1985; Blanchard and Kiyotaki 1987; Ball and Romer 1990).\footnote{The connection with Modigliani is also attested by the fact that from the mid-1980s to the second half of the 1990s this strand of the new Keynesian economics was developed and taught principally at MIT and by Modigliani’s former students. The central role attributed to market imperfections also bears the mark of Modigliani’s influence (see Hart 1982). In this regard, suffice it for us to recall that the results of Modigliani’s (1944) model are reaffirmed and strengthened in Modigliani (1963)), which assumes imperfect markets.} The result is that the unemployment equilibria derive from partial equilibrium models centered on the labor market, whose working is no longer influenced by the level of aggregate demand in the goods market but depends only on the maximization of expected profits at the efficiency wage (see, for example, Shapiro and Stiglitz 1984; and, by comparison, equations (2.5.)-(2.8. above), or else can be based on “non-adjustment”
in goods prices, which causes second-order efficiency losses for individual firms but first-order efficiency losses at the macroeconomic level (Akerlof and Yellen 1985).

A limit of this strain of New Keynesian economics is that the (positive or negative) exogenous shocks, which leads to the non-adjustments in goods prices, cannot be absorbed by adjustments in the money supply inasmuch as the latter is exogenously fixed. In other words, in the short run monetary policy again has “real” effects in that it serves to remove these exogenous constraints (see Blanchard 1990). Hence, the new Keynesian economics with market imperfections solve only a part of the problems bequeathed by the neoclassical synthesis. While they have the merit of rendering endogenous the nominal and real rigidities that were previously treated as exogenous, from the monetary standpoint they mark no substantial progress with respect to Modigliani (1944, 1963). To see this, we need only consider that money and monetary policy revert to being neutral in the long term.6

3. Credit rationing and unemployment

It is beyond our scope here to investigate in depth the above-mentioned limitations or other problematic aspects of the strand of new Keynesian economics with market imperfections and endogenous rigidities (for further discussion, see Messori 1999). It is sufficient to repeat that this approach cannot overcome the analytical fragility of the results of unemployment equilibrium reached by Modigliani (1944) and the neoclassical synthesis. Hence, it is legitimate to ask whether a more robust determination of unemployment equilibria might be based on the elimination of all nominal or real price rigidities and on the endogenization of financial constraints.7 As far as we

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6 The link with Modigliani’s analysis (1963, 1977) remains very strong. In Modigliani, the Keynesian equilibria and non-neutrality of money are ultimately short-term phenomena because they are tied to the elasticity of the various parameters and to the consequent velocity of adjustment of the different variables. The difference vis-à-vis Friedman’s monetarism therefore tends to boil down to the problem of how short is the short term, which is very important for economic policy but less for theory.

7 This is the path marked out by a second strand of the new Keynesian economics, that based on information asymmetries (see, for example, Stiglitz 1987; Greenwald and Stiglitz 1993). Besides arguing that price flexibility does
know, in a partial equilibrium context, economic theory has defined only one endogenous constraint in the quantity of monetary flows supplied in real terms, namely rationing in the credit market. It is thus appropriate to recall some stages of the related debate.

Economic theory has labored hard to prove that equilibria with quantity rationing not constrained to Pareto efficiency can be reached in the credit market. The key intuition was supplied by Keeton (1979, ch. 3) for the case of moral hazard (with hidden action) and was elaborated upon by Stiglitz and Weiss (1981), who extended it to the case of adverse selection: the terms of the debt contract negatively influence the behavior or the quality of borrowers. Therefore, increases in the interest rate or in the amount of collaterals required will induce borrowers to select the riskiest projects or else induce potential borrowers with less risky projects to leave the market, thereby increasing the likelihood of default of the remaining set of debt contracts. Consequently, above a certain interest rate (or collateral requirement) threshold, these negative effects tend to prevail over the obvious increment in banks’ expected return in the event of solvency (or default) of the borrowers. The curve of the average return that the bank expects to obtain from the set of debt contracts offered is bell-shaped with respect to the interest rate (or collateral required). Since it is derived from that curve, the credit supply curve reaches one or more maximums and can position itself — in the actual segment — below the traditional credit demand curve. In such a case, the equilibrium interest rate, set by the bank, is lower than the Walrasian notional rate, so that there is equilibrium credit rationing.

The contributions of Keeton and of Stiglitz and Weiss (hereinafter SW) are based on various restrictive assumptions. During the 1980s many authors asked whether the credit-rationing equilibrium was robust to a relaxation of these assumptions. It was stressed, for example, that banks could use variations in the interest rate and collateral requirements in order to discriminate projects

not imply full employment equilibria, this strand attaches importance to the quantity constraints in the financial markets (see also fn. 1 above). As will be seen in our analysis of credit rationing, however, it obtains its most interesting results in partial rather than general equilibrium models.
of differing risk but which for the banks were indistinguishable *ex ante* owing to information asymmetries. This prompted Bester (1987) to argue that a bank having both these instruments available would resort to equilibrium credit rationing only if limits in the amount of wealth that the borrowers could pledge as collateral prevented the realization of full-separating equilibria. And it prompted Besanko and Thakor (1987b) to show that rationing itself can be a necessary condition to obtain full-separating equilibria. SW (1992) were thus obligated to add to their basic model: introducing differences in borrowers’ wealth as a further element of information asymmetries suffered by banks, they prove that variable and endogenously determined interest rates and collateral requirements generate a multiplicity of equilibria, including (semi-)separating and rationing equilibria.

However, the results of SW (1981 and 1992) are not robust to a further relaxation of their restrictive assumptions: the admission of divisible rather than indivisible projects. In SW loan demand is of a predetermined amount (normalized to 1). If this demand is not fully satisfied, it falls to nil because the project to be financed is unfeasible with an investment smaller than 1. By contrast, Milde and Riley (1988) and other authors restore the more general hypothesis of divisible projects, which had been adopted by all the models of credit rationing in the first and unfruitful phase of the debate (see, for example, Hodgman 1960; Jaffee and Russell 1976): each borrower’s demand is a decreasing function of the interest rate, and if it proves to be in excess at the equilibrium interest rate, the actual amount of the loan is positive but constrained by bank supply. In the case of divisible projects, we will have credit rationing if: (i) at the market interest rate and other market terms of contract, some or all of the borrowers obtain an amount of credit that is positive but smaller than that demanded; (ii) the resulting equilibrium does not dominate, in terms of Pareto-efficiency, either the hypothetical Walrasian equilibrium or other possible non-rationing equilibria. This equilibrium condition is called type I credit rationing.\(^8\)

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\(^8\) In the case of indivisible contracts, instead, we will have what we call type II credit rationing if, given a set of borrowers that the bank evaluates *ex ante* with identical default risk, a part of this set obtains the full amount of credit
The problem — as Milde and Riley (1988) and others (e.g. Besanko and Thakor 1987a; Innes 1991) show — is that the examination of divisible projects weakens the possibility of credit rationing. By choosing the size of the project to be financed, firms give the bank information about their own degree of risk. In other words, they “signal” their specific quality, which is not immediately perceptible to the bank due to information asymmetries. It follows that, in the model of Milde and Riley (1988), separating equilibria are obtained in which, depending on the project’s production technology (multiplicative or additive), the high-quality firms have to accept over- or under-financing in order to distinguish themselves from those of low quality. Note too that: (i) the existence of separating contracts alone depends on the notion of equilibrium used; (ii) the standard debt contract is suboptimal. Signaling is costly and not always advantageous, since high-quality firms must sign debt contracts that make it too costly for low-quality firms to engage in imitative behavior, and it is imperfect, since the signal’s significance varies depending on the specific technology of the project to be financed. Hence, adopting a different concept of equilibrium, in Innes’s (1991) model both separating and aggregating equilibria are obtained. Given the model’s (technological) assumptions, the aggregating equilibrium implies underfinancing for the high-quality firms and overfinancing for the low-quality firms.

Dispensing with some technicalities, the results reached in the models with divisible projects can be summarized in two points: (1) type I credit rationing equilibria are attributable to separating equilibria, in which high-quality firms select the contracts with underfinancing in order to distinguish themselves from low-quality firms, or, if this is too costly, to aggregating equilibria, in which high-quality firms are underfinanced and low-quality ones overfinanced; (2) the emergence of one of these specific equilibria depends on ad hoc assumptions regarding the technology of the project to be financed and the degree of difference in quality between firms, so that the set of demanded while the other part (drawn at random) obtains no credit, even if it would be willing to accept the same terms of the contract or even more onerous ones.

In particular, replacing Wilson’s (1977) with Riley’s (1979).
equilibria under point (1) is reduced to a series of specific cases rather than multiple equilibria of one and the same model.

Points (1) and (2) show that the results reached by the literature on credit rationing in the first half of the 1990s were far from definitive. The type I credit rationing analysis, which is based on more general hypotheses regarding the characteristics of the projects financed, is less robust than the type II credit rationing analysis (see above, fn. 8). Accordingly, it would have been logical to expect the appearance of a number of different models positing projects of variable size in order to refine the analysis of type I credit rationing or underinvestment equilibria. Yet, apart from isolated instances (for example, Ardeni and Messori 1999 and 1996), the theoretical literature on equilibrium credit rationing has not pursued this path. Various contributions have been devoted to reiterating — along the lines of De Meza and Webb (1987) — that, notwithstanding the presence of liquidity constraints for specific categories of borrower, information asymmetries generally cause overinvestment rather than rationing (see De Meza and Webb 1999 and 2000; Webb 2000; De Meza 2002). Indeed, applying a similar approach to projects of variable size and carrying some aspect of the Milde and Riley (1988) model to the extreme consequences, it has even been suggested that type I credit rationing is more apparent than real, since it refers to an (inefficient) choice of underinvestment on the part of individual firms (see Bernhardt 2000). Regarding credit rationing in the strict sense, the most interesting theoretical and empirical contributions are perhaps to be found in the study of quantity financial constraints in underdeveloped countries (see, for example, Armendariz de Aghion and Gollier 2000).

Is there a plausible explanation for this turn in the literature? One is that with the start of the 1990s equilibrium credit rationing ceased to be one of the constituent elements of the credit channel. While Blinder (1987) still argued that the monetary policy transmission mechanism underpinned by credit rationing has a significant role because it poses quantity rather than price

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10 Inquiry into the liquidity constraints to which firms are subject during start-up is of particular interest. In this regard, see the works of Cressy (especially 2000), which hark back to the model of Evans and Jovanovic (1989).
constraints, Bernanke and Blinder (1988 and 1992) limited rationing to a possible but non-essential case of the credit channel, and Gertler and Gilchrist (1993) even sidelined that case in favor of more generic mechanisms of propagation of financial constraints. In short, the credit rationing models have been closed in the stockade of a pure theory devoid of policy relevance. This first explanation throws into further relief a second one, which poses what here is the important problem. Analyses of credit rationing are limited by their being confined within partial equilibrium models and usually not extendable to general equilibrium models. This is a serious limit, for it has hindered study of the possible links between credit rationing and the unemployment equilibrium.

Our thesis is that the quantity monetary constraints imposed by credit rationing can be credibly binding for the determination of the level of activity, and can therefore be a cause of unemployment, only if they are integrated into a general rather than a partial equilibrium model. A similar line of research is pursued by Fuerst (1994), but his ‘general equilibrium’ analysis is based on a number of ad hoc assumptions (see below, section 6). However, in our setting, when credit supply à la SW is inserted into the aggregate macroeconomic model of the neoclassical synthesis examined above (see 2.1.-2.10, Section 2), it can be seen that a quantity monetary constraint of this kind is not generally sufficient. In the case in question, an unemployment equilibrium is obtained only under a quite specific hypotheses and within the context of a situation that can be called a “deflationary gap”. In the following section we shall first demonstrate the validity of the foregoing statement and then describe the structure of a general model which, albeit by utilizing only type II credit rationing, reaches an unemployment equilibrium thanks to an endogenous quantity constraint in the credit market.

4. Novelty and structure of the model

11 Wasmer and Weil (2002) examine the relationship between the credit and the labor market but they do not take into account of the equilibrium in the goods markets; hence, they simply posit the simultaneous existence of credit rationing and unemployment.
Assume that in a partial equilibrium model à la SW (1981) banks collect the public’s financial savings and supply an equivalent amount of credit to finance firms’s investment demand. Given the level of income, the credit supply curve has a bell shape that depends on the non-monotonic behavior of the expected rate of return on these loans in the presence of adverse selection effects. As in the neoclassical synthesis model (cf. 2.1-2.10), the economywide investment \( I \) and saving \( S \) functions are defined in real terms (see Fig.1).

Figure 1

Let \( S' \) indicate the savings supply curve deriving from full-employment income. Assume that we start out from a credit rationing equilibrium at a less-than-full-employment level of income, in correspondence with the optimal loan interest rate \( r^\ast \) (curve \( S \)). Plainly, the disequilibrium in the labor market will push down the real wage rate. The interest rate and hence the notional expenditure for investment being equal, the level of activity and of real income will increase as a consequence of the advantage to firms of raising the employment level and of the proportionate increase in aggregate demand generated by the income multiplier. The latter effect stems from the reduction in the degree of credit rationing for investment demand, which at the optimal interest rate derives from the increase in the supply of savings due to the expansion in economic activity.

Clearly, in an “inflationary gap” at the optimal lending rate — i.e. with \( I(r^\ast) > S'(r^\ast) \) — the process described is bound to come to a halt only when the full-employment equilibrium position is reached. The only path open to verify the possibility of an unemployment equilibrium thus remains that of the situation of “deflationary gap”, i.e. when \( I(r^\ast) < S'(r^\ast) \) (see Fig. 1). But in that case as well we encounter nearly insuperable difficulties, because, in the impossibility of remunerating savings at the optimal rate of return, the interest rate will tend to decrease in the face of the excess supply of savings. This pushes the system toward the full employment equilibrium level of income.
at interest rate $r'$. We conclude from this that there will be a Keynesian unemployment equilibrium only in the very unlikely event that the deflationary gap is accompanied, in correspondence with the optimal interest rate $r^*$, by an equal slope of the investment and saving curves.\footnote{In this particular case, the rightward shift of the savings supply curve comes to a halt at the point of tangency with the investment demand curve, in correspondence with the optimal interest rate. Consequently, economic activity will be below the full-employment level.}

The foregoing analysis would appear to vitiate any attempt to strengthen the concept of unemployment equilibrium by introducing endogenous credit rationing and by eliminating all price and wage rigidities within the framework of a general model. It is our view, however, that the result reached is attributable to an oversimplified demand for credit. In the remainder of this paper we therefore present a simple general equilibrium model in which the decision of the banks to ration credit cannot be reabsorbed by adjustments in the level of loan demand and supply, and we demonstrate that the banks’ choice implies unemployment equilibria. Before proceeding in that direction, in the rest of this section we shall specify the characteristics of our aggregate general equilibrium model.

There are three groups of private agents (firms, households and banks) and one public agent (the central bank). All the private economic agents are risk-neutral. The temporal structure of the model is defined by two instants of time, $t=0$ and $t=1$, which delimit the period examined.

For the financial markets, we assume –in principle, without loss of generality – that no market in private or public securities exists; moreover, at $t=0$ there are no positive or negative stocks of monetary assets in the balance sheets of households and firms. This implies that at $t=0$ firms can finance their activities only if they obtain an adequate amount of credit from the banking system; and that during the period examined or at $t=1$ households can only hold the income they earn and their savings either in the form of currency or in that of bank deposits.\footnote{We also assume for now that there is only one type of bank deposit and that, at $t=1$, firms do not retain a part of the profits they have made but distribute these to the owner households. Note that the two assumptions are not strictly necessary for the validity of our analysis. Complicating the model slightly, it would be possible to suppose that households liquidate a variable share of their banking deposits (of various kind) and that firms have margins of self-financing.} The supply of fiat money
(i.e. currency) is the monopoly of the central bank, which transfers it to the banking system as liabilities. The banks hold reserves in the form of non-circulating legal tender. Apart from these reserves, the only assets in the financial markets are currency in circulation, bank deposits and bank loans. The credit channel is therefore the only significant transmission channel (see Bernanke 1983); in general, it can also depend on the private agents’ decisions regarding what proportion of their own incomes to hold in the form of cash.

At $t=0$ firms are endowed with a production technology and a specific, indivisible investment project. Their production inputs consist of capital goods and labor units. At $t=0$ they hold a given stock of capital goods, $K$, that cannot be modified by the simultaneous decisions concerning the investment project, and so to start production they must only acquire labor units. By the end of the period examined, at $t=1$, the production technology (production function) generates a deterministic quantity of goods: $y = f(N, K)$. As regards each investment project, the required quantity of the capital good — here assumed to be identical to the consumption good — is indivisible and equal to $X$. Net of what is consumed in current production, the realization of the given investment project increases the stock of capital goods that will be available to the firm at the start of the subsequent period. Owing to the uncertainty about future technological and market conditions, each firm’s investment project generates stochastic returns in subsequent periods. Specifically, following SW (1981), we assume that the future returns on the individual investments of the firms have the same expected value but different probability distributions, which depend on their different risk, according to the principle of the mean preserving spread.

In the economy there is an asymmetric distribution of information among the three groups of private agents (banks, households and firms). The degree of risk of each investment project is the private knowledge of the firm, which is endowed with it. In addition, each firm also knows, by definition, the common expected return (expressed in real terms) of the investments and, obviously, the production function. These two data are also common knowledge for the banks, which have a
specific “information technology”. Moreover, although they do not know the risk of the specific investment projects of each firm, the banks know the distribution of the risk of these projects within the population of firms. By contrast, households do not have access to the banks’ “information technology”\textsuperscript{14} and therefore know neither the characteristics of production nor the risk of the investment projects. This justifies the fact that savers find it advantageous to hold their savings in the form of bank deposits rather than using them to finance firms directly (see, for example, Diamond 1984). As at \( t=0 \), firms may not have liabilities toward households at \( t=1 \).

As mentioned, given the structure of the model, at \( t=0 \) firms can acquire the labor inputs they need in order to start production and can carry out their own specific, indivisible investment project only by borrowing from the banks. Bank financing of production is equal to the total of money wages paid out; bank financing of each of the decided investment projects is equal to the product of \( X \) and the unit price of the capital good.\textsuperscript{15}

The foregoing is sufficient to provide an initial representation of the working of the economic system. Our simple model is based on the opening of two markets (the credit market and the labor market) at \( t=0 \) and of one market (the goods market) at \( t=1 \). The choices of banks and firms in the credit and labor markets at \( t=0 \) depend on the expected equilibrium values in the goods market. But once the equilibrium in the first two markets has been determined, these expected values already become certain at \( t=0 \) because they are based on the deterministic output of production and on the amount of the indivisible investment projects actually decided by the firms and financed by the banks. Thus, as in the usual general equilibrium analyses, our model can be reduced to the solution of a system of equations in an instant of time.

Let us examine the working of the three markets more closely. The determination of equilibrium in the labor market depends on traditional criteria: as the real wage \((w/p)\) rises, the supply of labor

\textsuperscript{14} This is a standard assumption and can be justified in various ways. One of the most common explanations is that for the individual non-financial agent access to information is costly and the benefit derived becomes a “public good”; this explanation raises the well-know free-rider problem.
increases and the demand for labor decreases; the nominal wage rate is perfectly flexible and adjusts instantaneously to each variation in the general level of the prices of goods. Assuming that the price adjustments in the goods and credit markets accommodate the aggregate demand to the optimal notional supply, in the labor market a full employment equilibrium should always be achieved. On the other hand, in the goods market the equilibrium between aggregate demand and supply is ensured by the equality between realized saving and realized investment, making it possible to illustrate the working of this market with the Keynesian investment multiplier.

However, this traditional set-up is re-defined by the working of the credit market, which constitutes the pivotal point of our inquiry. Let us assume that central bank, by means of advances, has already supplied the banking system with the amount of fiat money (monetary base, $H$) that functions as a reserve and constitutes the liquidity constraint for credit activity. Following the classic representation of the process of the extension of bank credit through the opening of offsetting deposits, banks estimate the redeposit ratio based on their own share of the deposit market. This determines the individual bank’s potential supply of credit, which is equal to the product of a given credit multiplier and the overall amount of its liquidity.$^{16}$ This multiplier is a known function of both banks’ reserve ratio, which is exogenously set under policy rules, and of the public’s desired liquidity ratio relative to its bank deposits. As for the structure of the credit market, we assume, only partly following SW (1981), that the banks operate in a regime of monopolistic competition à la Bertrand in both the loans and the deposits market. Every bank sets its lending rates so as to maximize its expected rate of return. However, the competition of the other banks (already in the market or potential entrants) forces each bank to offer a deposit interest rate, $r_D$, that cancels out its expected profits on the loans.

$^{15}$ Recall that the capital good is indistinguishable here from the consumer good.

$^{16}$ It can be demonstrated that the individual banks’ potential credit multiplier is identical and independent of their shares of the deposit market even where these shares differ; see Cesaroni (2000).
For the financing of total wages, the deterministic and socially efficient nature of output implies that firms’ prospects of solvency at the equilibrium interest rate are identical and certain; given symmetric information as well, for the banks it is a question of offering a standard debt contract \( r_f \) that is short term and ensures certain returns.

The financing of the specific indivisible investment projects to which the individual firms have access is more complicated. Given the stochastic nature of the expected returns from each of these projects and the presence of socially inefficient projects, from the standpoint of risk the individual firms are borrowers of different quality. However all the indivisible investment projects have the same expected return and, given asymmetric information, appear identical \textit{ex ante} to the banks (see above). Under these assumptions and on the basis of the chosen ordering of the random variables (mean preserving spread), this means — with SW (1981) — that, in a partial equilibrium context, at the equilibrium contractual interest rate \( r_f^* \) the banks may find it advantageous to ration the total demand for investment financing, represented by an inverse function of \( r_f \). In fact, the standard long-term debt offered by the banks to finance the indivisible investment projects tends to trigger adverse selection effects. The increases in \( r_f \) above the minimum level will prompt the firms with the investment projects having the least risky distribution of random returns to leave the credit market earliest. The upshot is that above one or more critical thresholds of the interest rate \( r_f^* \) banks’ average expected return on the total financing of these investments will tend to decrease.

5. Credit rationing and unemployment equilibrium

The description of our model can be specified in formal terms. The assumption of a given number of identical banks simplifies the exposition, making it possible to relate the conditions of the single bank equilibrium directly to the aggregate variables. Note too that all the variables are expressed in real rather than monetary terms (see below).
In our context, aggregate supply and demand for credit are specified, respectively, by:

(5.1.) \[ L' = mm \cdot H \]

(5.2.) \[ L^d = I(r_i) + F(w, N, r_f), \]

where \( mm = \frac{1}{\chi + \tau} \) is the credit multiplier and where: \( \tau \) denotes the required ratio of banks’ reserves to deposits; \( \chi \) the households’ liquidity ratio relative to deposits; \( H \) the monetary base (in real terms) that the banks obtain from the central banks; \( I(r_i) \) firms’ aggregate fixed investment demand, which depends on the real interest rate on investment loans; \( F(w, N, r_f) \) the demand for advances of working capital, which depends on the real wage \( w = \frac{w}{p} \), on the planned level of employment and on the real interest rate on loans of this type, \( r_f \).

In particular, the function, which describes the demand for loans on the part of the set of firms for the purchase of new capital goods (i.e. total demand for financing for the indivisible investment projects), will be of the kind:

(5.3.) \[ I(r_i) = X \cdot \int_{X + (\tau r_i)} g(R) \cdot dR; \]

where \( R \) and \( g(R) \) indicate, respectively, the random rate of return on the investments in the case of success and the density function of the firms corresponding to that rate of return, and \( X \) indicates the given scale of investment common to all the firms.

As the interest rate on investment loans, \( r_i \), increases, the indivisible investments diminish because the total number of firms able to obtain a positive expected return on these investments decreases.

As to the total demand for loans to acquire working capital, in our set-up the entrepreneurs have no wealth and no alternative opportunities to use their business ability. As long as the overall
revenues of production are greater than \((1 + r_F) \cdot w_r \cdot N\), the demand for loans for working capital will not depend on the related interest rate\(^{17}\), \(r_F\), so that we can put

\[(5.4.) \quad F(w_r, N, r_F) = w_r \cdot N.\]

Let us now go further into the specification of overall credit demand and supply in the context of the general equilibrium of the labor, goods and credit markets. In the first place, since bank credit lines are extended through the opening of offsetting deposits, \(L^r = D\): the supply of credit is equal to the deposits — expressed in real terms — that agents desire to hold. In our model deposits are held for transaction purposes or as a form of financial wealth alternative to cash. In general, therefore, deposits will depend on the level of income and on the deposit interest rate, since the latter influences the volume of saving and/or the desired liquidity ratio. For example, making a distinction between deposits held for transaction purposes and deposits held as a form of financial wealth, which becomes important if the former have significant withdrawal costs, the households’ supply of deposits with the banking system could be expressed as:

\[D = (1 - \ell) \cdot (y - S) + S,\]

where \(y\) and \(S\) represent, respectively, the level of income and of real savings, and \(\ell\) represents desired liquidity for expected transactions, \((y - S)\).

On the basis of an analysis à la Baumol and Tobin, \(\ell\) can be considered as a function of three variables: income, the rate of remuneration of deposits, and withdrawal costs: \(\ell \equiv \ell(y, r_D, t_c)\). Note that, in general, \(\chi \neq \ell\), since \(\chi = \frac{\ell \cdot (1 - s(\cdot))}{1 - \ell \cdot (1 - s(\cdot))}\); therefore, \(\chi = \ell\) only when \(\ell = 0\).

Turning to the working of the labor market, we will have \(w_r \cdot N = w_r(y) \cdot N(y)\). This means that, by means of the monotonic function of the marginal productivity of labor, the equilibrium level of

\(^{17}\) The possible dependence of the demand for working capital on the short-term interest rate would only marginally complicate the analysis without invalidating our conclusions. See the appendices.
production determines the total wages demanded by firms. Lastly, the equilibrium in goods markets presupposes that realized investment is equal to saving. If all the investment projects were financed, we would therefore have:

\[ I(r_i) = S = s(r_D, y) \cdot y; \]

where \( s(r_D, y) \) represents the average propensity to save, which is a function of real income and of the real interest rate on deposits.

Just for the sake of simplifying our analysis, let us assume \( \ell = 0 \); in their transactions agents do not use currency.\(^{18}\) After making the appropriate substitutions, we can rewrite the total credit demand and supply functions as:

(5.1. bis) \[ L' = y \]

(5.2. bis) \[ L^d = I(r_i) + w_r(y) \cdot N(y). \]

Given the modeling adopted, we ask whether there can be an equilibrium with credit rationing for firms and an underemployment level of output can exist in the economic system considered. In particular, for the structure of the model adopted, the case that will be important is that of rationing of the financing of fixed investment.

We shall indicate with \( r^*_i \) the value of the interest rate on loans to finance the investment projects that, given the adverse selection effects, maximizes the expected rate of return on such loans, \( \rho(r_i) \) (see SW 1981); \( \rho(r_i, r_F) \) indicates the average rate of return that the bank expects to realize on the overall volume of loans. Competition among banks imposes:

(5.5.) \[ r_D = \rho(r_i, r_F). \]

\(^{18}\) In the more general case, in which agents also use currency in transactions, the substance of the demonstration presented in the appendices would still be valid. The possible complications would regard the functional form of \( A \) (see below).
We shall define $\varepsilon > 0$ as a variable measure of the rationing of firms' investment projects, i.e. as a measure of the demand for capital goods decided by the firms but not financed by the banks, so that the actual investment projects are equal to $I(r_I) - \varepsilon$. In correspondence with a given measure of rationing, the total supply and demand for credit, which are compatible with the equilibrium in the good market and with the firms' labor demand function, become:

\begin{align}
L' &= y(\varepsilon) \\
L^d &= I(r_I) + w_I(y(\varepsilon)) \cdot N(y(\varepsilon))
\end{align}

with

\begin{equation}
y(\varepsilon) = \frac{[I(r_I) - \varepsilon]}{s(r_D(\varepsilon), y(\varepsilon))}.
\end{equation}

In the credit market we have, moreover:

\begin{equation}
L' = L^d - \varepsilon.
\end{equation}

From (5.6.), (5.7.) and (5.9.), we obtain:

\begin{equation}
y(\varepsilon) - [I(r_I) - \varepsilon] = w_I(y(\varepsilon)) \cdot N(y(\varepsilon))
\end{equation}

that is to say, the credit supplied to production must be equal to its demand.

Assuming that there is a $\varepsilon > 0$, $\varepsilon \in (0, I(r_I))$, able to satisfy (5.10.) and such as to satisfy:

\begin{align}
\max & \quad r_D(\varepsilon) \\
y(\varepsilon) & < y^{FE},
\end{align}

where $y^{FE}$ indicates full-employment output;

then we will have a credit rationing and unemployment equilibrium.

Setting the lending rate for the investment projects at the optimal level, $r_I^*$, and putting:
it is possible to specify a set of “weak” assumptions that are sufficient to identify the equilibrium sought. The demonstration of the existence of this equilibrium is set out in detail in the appendices. Here, we need only recall that to prove the validity of conditions (5.10.) and (5.11.) it is sufficient that:

a) in \( \varepsilon = I(r_f^*) \), the slope of \( A \) have a modulus greater than that of the slope of \( B \);

b) functions \( A \) and \( B \) have an intersection \( \varepsilon^* \) in the interval \( (0, I(r_f^*)) \).

Assumptions a) and b) imply that, having assumed that \( \rho(r_f^*) > r_f(\varepsilon) \), \( \varepsilon^* \) corresponds to a maximum in the average rate of return on overall loans, \( \rho(r_f^*, r_f(\varepsilon)) \) (see below, and Appendix 2).

The analytical derivations of the expressions of \( \frac{\partial A}{\partial \varepsilon} \) and \( \frac{\partial B}{\partial \varepsilon} \) (see Appendix 1) and the shares of the two types of loan in the total supply of credit allow us to show the plausibility of the assumptions used, in that they are easily derivable from standard hypotheses regarding the form of the production function and the response of the average propensity to save to variations in credit rationing. In particular, it is necessary to assume that the marginal productivity of labor is not constant but decreasing.

The demonstration can be summarized graphically (Figure 2). If assumptions a) and b) are satisfied, we will have the following situation:

![Image](image-url)

Figure 2

Figure 2 illustrates the existence of a positive measure of rationing of fixed investment, \( \varepsilon^* \), that ensures equality between demand and supply of credit for working capital. In \( \varepsilon = I(r_f^*) \), we have
\[ A(\varepsilon) = B(\varepsilon) = 0 \]: the equilibrium level of aggregate demand and, hence, the level of planned production of the set of firms are nil. Now, it is possible to demonstrate that in the interval \((0, \varepsilon^*)\) the average rate of return on loans increases monotonically as \(\varepsilon\) decreases. Nevertheless, values of \(\varepsilon : \varepsilon < \varepsilon^*\) are not compatible with the equilibrium of the system, since they prevent the supply of credit for working capital from being sufficient to acquire the labor needed to achieve the equilibrium level of output in the goods market. Therefore, in the interval \((0, \varepsilon^*)\) function \(A\) indicates purely notional values: in that interval the equilibrium level of income, which gives rise to \(A\), cannot be produced. In other words, although profitable, values of \(\varepsilon : \varepsilon < \varepsilon^*\) are not feasible. One can also demonstrate that values of \(\varepsilon : \varepsilon > \varepsilon^*\), though compatible with the reaching of an equilibrium in the goods market, imply an average rate of return on loans that decreases monotonically as the degree of rationing increases. This outcome depends on the joint action of the excess supply of short-term loans and the reduction of long-term loans’ share in total loans. Hence, values of \(\varepsilon : \varepsilon > \varepsilon^*\), although feasible, are not profitable as they are incompatible with (5.11).

Ultimately, we can conclude that \(\varepsilon^*\) identifies de facto an absolute maximum of the average rate of return on loans, \(\rho(r^*_f, r^*_F(\varepsilon))\), since this measure of rationing in the financing of fixed investment constitutes a position of constrained optimum for the banks.

Completing our interpretation of Figure 2, it is appropriate to note that the level of output associated with \(\varepsilon = 0\) will depend on the relation between two parameters: the supply of savings corresponding to the full-employment level — that is, \(S^{FE} = s(\rho(r^*_f, r^*_F(0)), y^{FE}) \cdot y^{FE}\) — and the amount of investment decided by the set of firms at the optimal rate \(I(r^*_f)\), which represents the maximum degree of rationing. At \(\varepsilon = 0\), when \(I(r^*_f) < S^{FE}\), the level of activity will be below a full-employment level; when \(I(r^*_f) = S^{FE}\), a full-employment level will be reached; finally, when \(I(r^*_f) > S^{FE}\), there will be a notional level of activity higher than the full-employment level.

\[ As is show in Appendix 1, at that point both functions have a negative slope.\]
With reference to condition (5.12), we can thus state that in the case where $\varepsilon^* > 0$ when $I(r_i^*) \leq S^{FE}$, there will certainly be an unemployment equilibrium. On the other hand, where $\varepsilon^* > 0$ when $I(r_i^*) > S^{FE}$, it is not necessarily true that an unemployment equilibrium will be reached; this will happen only if $\varepsilon^*$ constitutes a sufficiently large measure of rationing.

Note that the foregoing demonstration of the existence of an unemployment general equilibrium was made exclusively in real terms. This implies that our analysis is based on a binding credit constraint but is compatible with the neutrality of money. In order to “close” our model, we have to determine the price level in accordance with the principles of the quantity theory of money.$^{20}$ Here, the key condition is the equilibrium between monetary base demand ($H^D$) and supply ($H^S$). In particular, assuming that the banks know the structural data of the economy and can therefore rationally predict the equilibrium values of the economic system (rational expectations hypothesis), the demand for monetary base in real terms can be made endogenous. This is equivalent to assuming that the commercial banks correctly estimate the equilibrium amount of real deposits, $D(\varepsilon^*)$. Since $L^s = D(\varepsilon^*)$, on the basis of (5.1) the commercial banks demand from the central bank a quantity of monetary base, $H^D$, equal to $\frac{1}{mm} \cdot p \cdot D(\varepsilon^*)$; where $p$ indicates the initial estimate of the price level and $\frac{1}{mm}$ is the reciprocal of the credit multiplier.

It follows that the equilibrium price level is determined by the nominal supply of monetary base established by the central bank, $\bar{H}^s$, the demand for which adjusts. That is to say:

(5.13) $\bar{H}^s = \frac{1}{mm} \cdot p^* \cdot D(\varepsilon^*)$.

With the general price level, $p^*$, established, nominal interest rates are determined by the sum of equilibrium real rates and the inflation expectations generated by $p^*$ given a certain elasticity.
6. Conclusions

This paper has explained how in a simple aggregate general equilibrium model, in which money wages and goods prices are, in principle, perfectly flexible, banks’ choices can determine credit rationing for fixed investment and an unemployment equilibrium. Key elements of our model are: the joint presence of credit for production and credit for investment demand; the operation of an adverse selection effect on the latter, which explains the advantage to banks of not adjusting the interest rate on investment loans in the face of excess demand; and the endogenous nature of the demand for monetary base in real terms.

Although it starts out from the basic elements of credit rationing models à la Stiglitz and Weiss (1981), our model has significant new features, adopting a general equilibrium rather than a partial equilibrium approach and including a dual demand for credit (for working capital and for fixed investment) rather than a single demand for financing for a given project. For the problem we have addressed, the graphic analysis of SW (1981) can be used to examine the shifts in the saving and investment curves following variations in the level of aggregate output caused by the flexibility of real wages; however, except in one utterly exceptional case, these shifts do not prevent wage and price flexibility from absorbing unemployment and leading to full employment equilibria (see above, section 4). On the other hand, in our general equilibrium model the presence of demand for credit for current production explains the existence of an optimal amount of credit rationing of fixed investment and consequently determines an unemployment equilibrium in the labor market. In fact, the banks must brake the expansion of credit for investment (i.e. prevent rationing from falling below a level $\varepsilon^*$) because otherwise the funds available to finance production would become insufficient. In this regard, it is worth stressing an important property of our model not found in the partial equilibrium model of SW (1981). In the latter, the shortage of loanable funds — in

\textsuperscript{20} Needless to say, this does not mean that we adhere to the quantity theory of money. It is only a consequence of the
correspondence with the optimal interest rate, \( r_i^* \) — is an exogenous assumption, whereas in our case the shortage is endogenous.

Fuerst (1994) attempts to offer a general equilibrium view of credit rationing. As far as we know, this is the only model where a credit market co-exists with markets for goods and labor; however, the economic analysis still suffers from some serious limitations. First, Fuerst’s “general equilibrium” is based on real wages, the stock of real money and exogenously given interest rates. Second, Fuerst offers no economic rationale for credit rationing in general equilibrium but simply posits the existence of type II credit rationing for an arbitrarily given proportion of firms and goes on to assert that this rationing is compatible with the markets equilibrium conditions when unknown “suitable restrictions” are imposed on the growth rate of the exogenous money supply.\(^{21}\) Hence, credit rationing remains unexplained and, quite obviously, its presence depends on the exogenous constraints imposed on the growth rate of money supply.

Apart from the crucial matter of the credit market, our aggregate general equilibrium model is very similar to that developed by the neoclassical synthesis in re-interpreting Keynes’s main conclusions. It can also be linked to disequilibrium theory and to some contributions in the more recent literature of the new Keynesian economics, based on nominal and real rigidities. The chief differences lie in the role played by money and bank credit; in particular, the latter is virtually missing in both these other strands.

In our model all the main variables are specified in real terms; thanks to the assumption of rational expectations, the banks are able to calculate the demand for monetary base in real terms, which corresponds to the equilibrium determined by the “optimal” measure of credit rationing. It follows that the system is in the real equilibrium position with a price level (and price expectations) model’s real structure and corresponds, moreover, to the most unfavorable case for the thesis we want to prove.

\(^{21}\) Fuerst gives a partial equilibrium explanation for type II credit rationing which is very similar to that of Stiglitz and Weiss (1981), whereas his so-called “rationing on the intensive margin” is not a form of credit rationing at all. Moreover, even Fuerst’s type II credit rationing is not well founded for it could be easily proved that the demonstration of proposition 1 case 3 (pp. 433–4) is wrong.
determined by the quantity of monetary base in nominal terms, which is managed and supplied by
the central bank; and this equilibrium position is independent of the actual quantity of monetary
base. The endogenous nature of the demand for monetary base in real terms distinguishes our
contribution from those of the neoclassical synthesis and from some general equilibrium models of
the new Keynesian economics (see Blanchard and Kiyotaki 1987). In the latter models, once the
presence of rigidities in money prices is given, the fact that money supply is exogenous allows an
expansionary monetary policy to guide the system toward an equilibrium position at a higher level
of employment; in our approach, by contrast, it is the combination of a constraint à la Clower and
the particular working of the credit market that justify the results obtained, that is credit rationing
and unemployment equilibria. In other words, the role of money as a medium of exchange is crucial
to justify our results; by contrast, in the new Keynesian models based on market imperfections and
endogenous rigidities, money is a non-produced good which plays the role of unit of account and
can be hoarded.

Lastly, we underscore a characteristic of the financial constraint obtained that distinguishes it
further from the traditional one of the neoclassical synthesis: the liquidity trap. Because of the
presence of credit rationing for investment demand, our constraint can act as an obstacle to the
attainment of the full-employment position even when there can be no liquidity trap, that is to say
when the interest rate on investment loans is at a level such as to generate inflationary gap
conditions \( I(r_j^*) \geq S^{FE} \) in correspondence with nil rationing.\(^{22}\) As we already remarked in
discussing the impossibility of such an outcome in the model of SW (1981), this is due to the
decisive role played by credit for production and the consequent complete endogeneity of the
shortage of credit.
APPENDICES

Appendix 1  The behavior of the equilibrium production-credit demand and supply functions

1.1. The derivative of the two functions

To begin with, we obtain the partial derivative of the production-credit supply function,

\[ A(\varepsilon) = y(\varepsilon) - [I(r^*) - \varepsilon]. \]

We immediately have:

\[ \frac{\partial A}{\partial \varepsilon} = \frac{dy}{d\varepsilon} + 1. \tag{A1.1.} \]

Calculating the partial derivative of the production-credit demand function,

\[ B(\varepsilon) = w_\tau(y(\varepsilon)) \cdot N(y(\varepsilon)), \]

is more complex. The labor market equilibrium condition permits us to state that the real wage is equal to the marginal productivity of labor:

\[ w_\tau = f'(N); \]

so that

\[ w_\tau \cdot N = f'(N) \cdot N, \text{ with } N = f^{-1}(y(\varepsilon)). \]

The invertibility of the function of the marginal productivity of labor, \( f'(N) > 0 \), implies that the employment level can be expressed as a function of the equilibrium level of aggregate demand.

Further, we have:

\[ \frac{\partial N}{\partial \varepsilon} = (f^{-1})' \cdot \frac{dy}{d\varepsilon}. \]

On this basis, deriving the new expression of total wages with respect to \( \varepsilon \) and simplifying, we obtain:

\[ \frac{\partial B}{\partial \varepsilon} = \frac{\partial (w_\tau \cdot N)}{\partial \varepsilon} = \left[ \frac{f''(N(\cdot)) \cdot N(\cdot)}{f'(N(\cdot))} + 1 \right] \cdot \frac{dy}{d\varepsilon}. \tag{A.1.2.} \]

Expression (A.1.2.), i.e. the behavior of the first derivative of function \( B(\varepsilon) \) in the \((B, \varepsilon)\) plane, depends on the product of the derivative of the equilibrium production level with respect to the

\[ \text{Obviously, to obtain this outcome the equilibrium measure of credit rationing will have to be sufficiently big (see above, section 5).} \]
measure of rationing and a factor equal to one plus the elasticity of marginal labor productivity to employment. In a standard case of macroeconomic modeling, that with a production function with constant elasticity of marginal productivity, the first derivative of \( B(\varepsilon) \) will coincide — but for a multiplicative constant — with the derivative of the equilibrium level of production with respect to \( \varepsilon \). For example, in the case of constant marginal productivity and in that of a Cobb-Douglas production function (constant returns to scale) we will have, respectively:

\[
(\text{A.1.3.}) \quad \frac{\partial B}{\partial \varepsilon} = \frac{\partial (w, \cdot N)}{\partial \varepsilon} = \begin{cases} 
\frac{dy}{d\varepsilon}, \\
(1-\alpha) \cdot \frac{dy}{d\varepsilon}
\end{cases}
\]

where, in the case of the Cobb-Douglas, \( 1-\alpha<1 \) indicates labor’s share of income and \( -\alpha \) denotes the constant elasticity of the function of marginal labor productivity with respect to employment.

1.2. The relative slope of the two functions

We saw above that the behavior of functions \( A \) and \( B \) depends on the derivative of equilibrium income with respect to the measure of rationing of fixed investment, \( \frac{dy}{d\varepsilon} \). In order to specify the determinants of these two functions, it is therefore necessary to develop the derivation of the implicit expression \( y(\varepsilon) \). We refer to function (5.8.) of the text (see section 5). Taking the total differential of that function with respect to \( \varepsilon \), setting it equal to zero and rearranging, we obtain

\[
(\text{A.1.4.}) \quad \frac{dy}{d\varepsilon} = -\frac{1}{s(.) + \frac{\partial s(.)}{\partial y} \cdot y(\varepsilon)} \cdot \frac{y(\varepsilon) \cdot \frac{\partial s(.)}{\partial r_p(\varepsilon)} \cdot \frac{\partial r_p(\varepsilon)}{\partial \varepsilon}}{s(.) + \frac{\partial s(.)}{\partial y} \cdot y(\varepsilon)}.
\]

Expression (A.1.4.) allows us to evaluate the plausibility of the two assumptions, (a) and (b), which were presented as sufficient conditions for determining the equilibrium measure of rationing, \( \varepsilon^* \), in the text (see section 5). Recall that on the basis of assumption (a), at \( \varepsilon = I(r_p^*) \) the (negative)
slope of function $A$ must have a modulus greater than that of the (negative) slope of function $B$. Also, recall that on the basis of (b), in the interval $(0, I(r^*_i))$ there must be an intersection between the two functions; given assumption (a), this requires that as $\varepsilon$ decreases the negative slope of function $A$ becomes less pronounced with respect to that of function $B$ for a sufficiently large range of values.

To demonstrate that assumptions (a) and (b) are plausible, note that when $\varepsilon = I(r^*_i)$ — i.e., at the point where there is a total rationing in the financing of fixed investment — we have $y(\varepsilon) = 0$ and, consequently, $A(I(r^*_i)) = B(I(r^*_i)) = 0$ and $\frac{dy}{d\varepsilon} = -\frac{1}{s(.)}$.

For (a) to be satisfied, at the point of maximum rationing of investment, we must have:

\[(A.1.5.) \quad \left| \frac{dy}{d\varepsilon} \right| + 1 > \left| (1-\alpha) \cdot \frac{dy}{d\varepsilon} \right|,\]

where: the left-hand member and the right-hand member represent the moduli of the partial derivative of function $A$ and function $B$, respectively.

Observe that at $\varepsilon = I(r^*_i)$ the partial derivatives of the two functions are negative, provided that the average propensity to save is a positive variable ranging between zero and one. If $0 < s(.) < 1$, expression (A.1.5.) reduces to: $\frac{dy}{d\varepsilon} < -\frac{1}{\alpha}$; and, when $\varepsilon = I(r^*_i)$, the latter expression becomes:

\[(A.1.6.) \quad s(.) < \alpha.\]

Empirical evidence shows that the average propensity to save tends to be very low in correspondence with low levels of income. Therefore, condition (A.1.6.), is not particularly restrictive.

It should also be noted that (A.1.6.) cannot be valid in the case of constant marginal productivity of labor, corresponding parametrically to $\alpha = 0$. The conclusion is that the analysis of the effects of
credit rationing on the unemployment equilibrium, analyzed in the text (see sections 4 and 5), is valid only if the production function is characterized by decreasing marginal productivity of labor.

However, in order to demonstrate that our analysis is valid in the case of production with decreasing marginal productivity of labor, we must still prove that assumption (b) is satisfied under not particularly restrictive conditions. Now it is generally reasonable to assume $\frac{dy}{d\varepsilon} < 0$ \forall \varepsilon < I(r^*_{i}) : since we expect the increase in the degree of rationing of fixed investment to reduce the level of equilibrium income, the slope of function $B$ will be negative everywhere. At this point we can distinguish two cases, both, however, leading to the same conclusion.

Concerning the first case, for values of $\varepsilon$ that are not too distant from $I(r^*_{i})$, the slope of function $A$ is certain to remain negative: the assumption that its slope is smaller than that of function $B$ will amount to assuming that — going to values of $\varepsilon < I(r^*_{i})$ — expression (A1.4) inverts the sign of inequality, i.e. that for a certain interval of values we have

$$\frac{dy}{d\varepsilon} + 1 < (1-\alpha) \frac{dy}{d\varepsilon}$$

(A.1.7.) is satisfied if, in modulus, the slope of function $A$ becomes smaller than that of function $B$. That is to say, (A.1.7) requires that:

$$\frac{dy}{d\varepsilon} > -\frac{1}{\alpha}$$

(A.1.8.)

We have the second case when, for values of $\varepsilon$ that are sufficiently distant from $I(r^*_{i})$, the slope of $A$ possibly becomes positive; in this case, assuming that the slope is smaller than that of $B$ will amount to supposing

$$\frac{dy}{d\varepsilon} + 1 > (1-\alpha) \frac{dy}{d\varepsilon}$$

(A1.9)

i.e. a condition that reproduces (A1.8) as a result.
Hence in both cases assumption b) will be satisfied if there is a sufficiently large range of values of $\varepsilon: \varepsilon < I(r_j^*)$, so that assumption (A1.5), valid at $\varepsilon = I(r_j^*)$, is reversed in (A1.8).

Examining the expression of $\frac{dy}{d\varepsilon}$, provided by (A.1.4.), for $y(\varepsilon) > 0$ it will be seen that there is no obstacle to the validity of the condition made necessary by assumption (b). In the shift from $y(\varepsilon) = 0$ to positive values of income ($\varepsilon < I(r_j^*)$) two factors will operate that will reduce the magnitude of the modulus of the derivative of equilibrium income with respect to the measure of rationing. First, there will be an increase in the denominator of the first addend of the expression (having a negative sign) due to the rise in the average propensity to save as income grows. Moreover, a second addend will emerge that will have a positive sign as long as $\frac{\partial r_d(\varepsilon)}{\partial \varepsilon}$ is negative and $\frac{\partial s(.)}{\partial r_d}$ positive. If these two effects are strong enough to verify (A1.8) along a sufficiently large range of values of $\varepsilon$, then assumption b) is satisfied and the two functions will have the required intersection. Utilizing (A.1.4.), we can rewrite condition (A1.8) as:

$$y(\varepsilon) \left[ -\frac{1}{\alpha} \left( s(.) + \frac{\partial s(.)}{\partial y} \cdot y(\varepsilon) \right) + \frac{\partial s(.)}{\partial r_d} \cdot \frac{\partial r_d(\varepsilon)}{\partial \varepsilon} \right] < -1.$$ 

The above observations allow us to better characterize the form of the production-credit demand function, $B$, given that its partial derivative is equal to $(1-\alpha) \cdot \frac{dy}{d\varepsilon}$ - that is, to a constant fraction of $\frac{dy}{d\varepsilon}$, which is negative. Starting from $\varepsilon = I(r_j^*)$, the decrease in the modulus of the first addend of the expression of $\frac{dy}{d\varepsilon}$ contributes to making the behavior of function $B$ concave as $\varepsilon$ decreases; as $\varepsilon$ diminishes further, the contribution will plainly become linear, if the average propensity to save becomes constant. Moreover, in the immediate vicinity of $\varepsilon = I(r_j^*)$ the concavity is strengthened by the appearance of a second positive addend (see A.1.3). The subsequent variations in the
magnitude of this addend, as $\epsilon$ decreases, will either accentuate the concavity (in the case of increases) or else will make a convex contribution to the behavior of function $B$. (in the case of decreases). In any case, we note that the denominator of the second addend is the same as that of the first, and that if $\frac{dy}{d\epsilon} < 0$ then the absolute value of the numerator of the second addend must be smaller than one: $\left| y(\epsilon) \cdot \frac{\partial s(.)}{\partial r_D} \cdot \frac{\partial r_D(\epsilon)}{\partial \epsilon} \right| < 1$.

Now, in these conditions — but in the particular case in which the numerator decreases too rapidly — we can demonstrate that the increase in the two addends’ common denominator implies that the variations in the first term dominate those of the second with respect to their order of magnitude. Hence, even if there were decreases in the magnitude of the second positive addend, which - as we know - act in favor of convexity, the behavior of the curve will normally be the concave one impressed by the first term.

We can conclude that, starting from the upper limit of the interval $[0, l(r_i^+)]$, function $B(\epsilon)$ has a negative slope and that its behavior is first concave and subsequently concave or possibly linear (or even slightly convex as a consequence of the second addend). On the other hand, we must take into account that the behavior of the function $y(\epsilon)$, on whose basis we determined the shape of function $B$, will have a more accentuated effect on the behavior of the supply function, $A$, since $\frac{\partial A}{\partial \epsilon} = \frac{dy}{d\epsilon} + 1$, whereas in the case of $B$ the effect was cushioned by the multiplicative coefficient $(1 - \alpha) < 1$.

1.3. Q.E.D.

---

23 In addition to the possibility that we have described, there is another point to make regarding the second addend of expression (A1.3). Note that as income increases the derivative of the average propensity with respect to the interest rate on deposits could become nil and then negative, owing to the possible weakening of the substitution effect between
Therefore, we can conclude that, in the case of production with decreasing marginal productivity of labor, assumptions (a) and (b) are satisfied under not particularly restrictive conditions. As already illustrated in the text (section 5, Figure 2), the result obtained can be translated into graphic terms. The outcome is a figure analogous to Figure 2 in the text:

Figure A.1

Appendix 2 The average rate of return on bank loans

The average rate of return on total bank loans, \( \rho(r^*_i, r^*_F) = r_D \), plays a significant role both in the derivation of the shape of functions A and B and in demonstrating that it is not advantageous for banks to increase investment credit rationing with respect to \( \varepsilon^* \), thereby avoiding the emergence of an economically trivial equilibrium with nil activity (\( \varepsilon = I(r^*_i) \)).

In both cases we supposed that \( \frac{\partial \rho(r^*_i, r^*_F)}{\partial \varepsilon} = \frac{\partial r_D}{\partial \varepsilon} < 0 \), i.e. that the average rate of return on loans (equal to the deposit rate in the competitive equilibrium) increased as the measure of rationing decreased.\(^{24}\)

Let us now consider Figure A.1. For values of \( \varepsilon \) included in \( \varepsilon^*, I(r^*_i) \), given the presence of an excess supply, the weight of short-term loans in the determination of the average rate of return on total loans will be determined by the demand, while the weight of long-term loans will depend on the supply, given the existence of an excess demand. In symbols, we will have

---

\(^{24}\) In the first case, recall that the hypothesis applies to the entire domain of the measure of rationing.
\[
\rho(r^*_f, r_f) = r_f(\varepsilon) \cdot \frac{w_r \cdot N}{y} + \rho(r^*_f) \cdot \frac{I(r^*_f) - \varepsilon}{y},
\]

since the supply of loans is equal to deposits, which coincide with income.

In the case of the Cobb-Douglas production function that we examined and considering the goods market equilibrium condition (5.8), the preceding expression will become

\[
\rho(r^*_f, r_f) = r_f(\varepsilon) \cdot (1 - \alpha) + \rho(r^*_f) \cdot s(.)
\]

since \((1 - \alpha)\) is the labor’s share — constant — and \(s(.)\) the average propensity to save.

Now, starting from \(\varepsilon = \varepsilon^*\), as \(\varepsilon\) increases the short-term lending rate, \(r_f\), decreases, given the presence of an excess of supply \((\frac{dr_f}{d\varepsilon} < 0)\), and the average propensity to save will also decrease; on the other hand the average rate of return on investment loans, \(\rho(r^*_f)\), is given, notwithstanding the existence of an excess of demand, due to the adverse selection effect. It follows that as the rationing of fixed investment increases, the average return on loans decreases, i.e. that

\[
\frac{\partial \rho(r^*_f, r_f)}{\partial \varepsilon} < 0 \quad \forall \varepsilon \in [\varepsilon^*, I(r^*_f))
\]

For values of \(\varepsilon \in [0, \varepsilon^*]\), the functional form that determines the average return on loans will change as a consequence of the shift from an excess supply to an excess demand for short-term loans. Now the relative weights of the two different forms of loans will be entirely determined by the characteristics of the supply of credit. We will have

\[
\rho(r^*_f, r_f) = r_f(\varepsilon) \cdot (1 - s(.)) + \rho(r^*_f) \cdot s(.)
\]
In this case, starting from $\varepsilon = \varepsilon^*$, as $\varepsilon$ decreases, along with the increase in the proportion of long-term loans due to the increase in the average propensity to save, the weights of the linear combination will change to the same degree, albeit in opposite directions. Since we normally have $\rho(r^*_f) > r_p(\varepsilon)$ — and since the short-term interest rate is increasing owing to the existence of an excess demand ($\frac{dr_F}{d\varepsilon} < 0$), the average rate of return on loans will increase as the degree of rationing decreases: $\frac{d\rho(r^*_f, r_p)}{d\varepsilon} < 0 \forall \varepsilon \in (0, \varepsilon^*]$. 

Therefore we have proved what we assumed in appendix 1, i.e. that

$$\frac{d\rho(r^*_f, r_p)}{d\varepsilon} \equiv \frac{dr_p}{d\varepsilon} < 0 \forall \varepsilon \in (0, I(r^*_f)].$$
References


